

TWO TOPICS IN ELEMENTARY PARTICLE PHYSICS:

- 1) THE REACTION $\gamma + N \rightarrow \pi + N$ AT HIGH ENERGIES
- 2) K LEPTONIC DECAY AND PARTIALLY CONSERVED CURRENTS

Thesis by
George Zweig

In Partial Fulfillment of the Requirements
For the Degree of
Doctor of Philosophy

California Institute of Technology
Pasadena, California

1964

ACKNOWLEDGMENTS

I would like to thank my advisors, Dr. Feynman, Dr. Gell-Mann, and Dr. Tollestrup who through their example teach ways of thinking that do not always come naturally to everybody.

I am also grateful to Dr. Frautschi and Dr. Walker for many interesting conversations.

Mrs. Beatrice Hall, Miss Cherry Carter, and my wife Audrey are to be thanked for their encouragement and typing of this thesis.

The financial support of the U. S. Atomic Energy Commission and the IBM Corporation is gratefully acknowledged.

ABSTRACT

$$1. \quad \gamma + N \rightarrow \pi + N$$

The process $\gamma + N \rightarrow \pi + N$ is studied at high energies in both the forward and backward directions. The helicity formalism is used. Contributions from the ρ , ω , ϕ , and π trajectories in the t channel and from the $N_{3/2}^*$, $N_{1/2}$, and $N_{1/2}^*$ trajectories in \bar{s} channel are included. Polarization formulae for the final state nucleon are given. If we may neglect cuts in the angular momentum plane, then at high energies for momentum transfers $-t \gg \mu_\pi^2$ the ρ trajectory should dominate charged pion photoproduction in the forward direction. A crude estimate of the cross section yields $\frac{d\sigma}{dt} \sim s^{2j_\rho(0) - 2} \approx \frac{1}{s}$. However, the pion trajectory is expected to be important for small momentum transfers up to very high energies. For forward neutral pion photoproduction both the ρ and ω trajectories should be important. The cross section is estimated to be $\frac{d\sigma}{dt} \sim \frac{1}{s}$. For photoproduction in the backward direction, the situation is much more complex with at least three trajectories contributing to $\frac{d\sigma}{ds}$. However, since the same set of trajectories are to be used in πN scattering, backward πN scattering and backward photoproduction are expected to have the same energy dependence. Assuming that the $N_{3/2}^*$ trajectory dominates with $j_{N_{3/2}^*}(0) \approx 0$, we obtain

$$\frac{d\sigma}{ds} \sim s^{2j_{N_{3/2}^*}(0) - 2} \approx \frac{1}{s^2}.$$

ABSTRACT (continued)

2. K Leptonic Decay and Partially Conserved Currents

An operational definition for the partial conservation of the strangeness changing vector current is given and applied to leptonic K^+ and K_2^0 decay. The K^* resonance is explicitly included in the calculation and quantitative agreement with experiment is obtained. A detailed comparison with the K^+ data of Brown et. al. and Dobbs et. al. is given. Because of rapid variations of a form factor, it is found that the data of these two groups are not in contradiction. From the K_2^0 experiment of Luers et. al., $I = 1/2$ and $3/2$ currents are seen to exist. $\Lambda\beta$ -decay is briefly considered. It is found that an explanation for the slowness of K leptonic decay and the vector part of $\Lambda\beta$ -decay may be connected with the partial conservation of the strangeness changing vector current.

TABLE OF CONTENTS

<u>Preface</u>	i - viii
 <u>$\gamma + N \rightarrow \pi + N$</u>	 1
I Introduction	1
II Trajectories in the t-Channel - Forward Photoproduction	4
A. The ω Trajectory	4
B. The ϕ and ρ Trajectories	12
C. The π Trajectory	12
D. Polarization of the Recoil Nucleon	17
III Trajectories in the \bar{s} Channel-Backward Photoproduction	19
A. The N Trajectory	21
B. The $N_{3/2^+}^*$ and $N_{1/2^-}^*$ Trajectories	23
IV Conclusions	25
 Appendix	 33
I The T Matrix and the Invariant Amplitudes	33
II The Helicity Amplitudes	35
III The t-Channel: $\gamma + \pi \rightarrow N + \bar{N}$	36
A. The π Trajectory	40
B. The ρ Trajectory	42
C. The ω and ϕ Trajectories	44
IV The \bar{s} Channel: $\gamma + \bar{N} \rightarrow \pi + \bar{N}$	45
A. The N Trajectory	45
B. The $N_{3/2^+}^*$ and $N_{1/2^-}^*$ Trajectories	49

	<u>K Leptonic Decay and Partially Conserved Currents</u>	51
I	Determination of a Theory for Leptonic K Decay	51
II	Predictions and Experimental Confirmations of the Theory	54
	Figure Captions for K Leptonic Decay	60
	Figures for K Leptonic Decay	63
	<u>References</u>	74

Preface

This thesis is divided into two sections, the first dealing with the process $\gamma + N \rightarrow \pi + N$ where the incident γ ray is very energetic ($\gtrsim 5$ BeV in laboratory system). The second section is concerned with another, entirely unrelated problem, that of trying to understand certain aspects of the β decay of strange particles where the strangeness of the strongly interacting particles changes by one unit; for example, the decays:

$$K^+ \rightarrow \pi^0 + \mu^+ + \nu$$

$$\Lambda \rightarrow p + e + \nu$$

$$\Xi^- \rightarrow \Lambda + e + \nu$$

In this preface we will briefly indicate the interest and significance of these two problems. We begin with a few introductory remarks concerning the photoproduction section of this thesis.

During the past year and a half much effort has been spent in an attempt to understand the behavior of cross-sections at high energies, and to correlate this behavior with the many particles and resonances that have now been discovered. Standard perturbation theory in its lowest order as well as the dispersion theory were unable to account for the experimentally measured total cross sections for πN or NN scattering which seemed to approach constants at high energies. Because of the large couplings involved, perturbation theory was not expected to work. Dispersion theory also had difficulties because of its inability to handle multi-particle final states. The usual pole approximations that made use of single particle or resonance intermediate states were certainly incorrect since they gave the wrong energy dependence to the total cross-sections.

For example, in πN or NN scattering, the exchange of a particle of spin j gives rise to a total cross section σ_T which goes like E^{2j-2} where E is the energy of the incident π or N in the center of mass system. If the exchanged particle were a high spin resonance, then σ_T would become arbitrarily large as E increased, contradicting experiment.

In a natural extension of the work of Regge, Chew and Frautschi proposed a way of avoiding this difficulty. Their idea was to exploit the "substitution law" which, for example, states in the case of πN elastic scattering that $\pi + N \rightarrow \pi + N$ and $\pi + \bar{\pi} \rightarrow N + \bar{N}$ are described by the same amplitude for different values of its arguments. More precisely, by analytically continuing the amplitude for $\pi + \bar{\pi} \rightarrow N + \bar{N}$ to large $\cos \theta_t$ (θ_t is the scattering angle for this process) they obtained the scattering amplitude for $\pi N \rightarrow \pi N$ at large energies and small scattering angles. Then by using the optical theorem, they related the forward πN elastic scattering amplitude to the total πN cross-section.

The work of Regge indicated how this analytic continuation to large $\cos \theta_t$ might be accomplished. The analytic continuation was not straightforward because the partial wave analysis for $\pi\bar{\pi} \rightarrow N\bar{N}$, usually applied only for $|\cos \theta_t| \leq 1$, does not converge for large $\cos \theta_t$. Regge's idea was to consider the angular momentum J as a complex variable, to replace the partial wave sum
$$\sum_J (2J+1) f_J P_J(\cos \theta_t)$$
 by a contour integral in the J plane, deform the contour, and in so doing obtain a sum that converges for large $\cos \theta_t$. This sum consists of contributions from poles and cuts in

the J plane that were picked up in deforming the contour. A pole in the J plane (called a Regge pole) corresponds to the exchange of a whole family (trajectory) of particles having identical quantum numbers except for spin. The position of the pole will be a function of the momentum transfer t .

Chew and Frautschi found the contribution to $\frac{d\sigma}{d\Omega}$ from a trajectory of particles to be given by $\frac{d\sigma}{d\Omega} = f(t) E^{4j(t)-2}$ where $j(t)$ indicates the position of the pole in the j plane and $f(t)$ is some unknown function independent of E . With the help of the optical theorem they obtained $\sigma_T \sim E^{2j(0)-2}$. They then suggested that $j(0)$ should be viewed as a phenomenological parameter with which to fit the experimentally measured σ_T . The hope was that at some future date a more refined theory would predict $j(0)$.

How many trajectories are there? There is a different trajectory for every known scalar and vector meson, and many more trajectories for the baryons and baryon resonances. Although far from conclusive, it seems that in many cases the baryon trajectories contain two or perhaps even three known particles or resonances. In addition, the existence of a trajectory (called the Pomanchuk trajectory P) with $j_P(0) = 1$ has been conjectured in order to explain the nearly constant high energy πN and NN scattering cross-sections ($\sigma_T \sim E^{2j_P(0)-2} = \text{constant}$). Finally, in order to obtain one more phenomenological parameter, another trajectory or cut called P' has been introduced into the formalism. If P' is a trajectory, then like the P , the particles contained on it have still not been found experimentally.

Although there are many trajectories and consequently many phenomenological parameters $j(0)$, each trajectory appears in different scattering processes. For example, the ρ is present in: $\pi N \rightarrow \pi N$, $NN \rightarrow NN$, $\gamma + N \rightarrow \pi + N, \dots$ By studying the high energy behavior of many different reactions one is able to check the assumption that scattering amplitudes take on relatively simple forms at high energies if expressed in terms of their singularities in the complex J plane.

The procedure we have outlined above has the advantage of supplying some kind of classification scheme for the newly discovered resonances, grouping them into trajectories, and associating with these trajectories certain parameters which may be used to fit the results of high energy scattering experiments. By observing cross-sections for many different processes, a check on the theory may be obtained. Unfortunately, the validity and usefulness of this procedure is still uncertain. For example, the importance of cuts in the J plane is not yet understood. Nevertheless, the theory is young and does offer both interest and promise.

In the near future with the completion of the electron accelerators at Cambridge, Hamburg, and Stanford the process $\gamma + N \rightarrow \pi + N$ will be studied at high energies. It would be interesting to correlate this process with other reactions like πN , KN , and NN scattering. The "Regge pole" approximation scheme, where one considers singularities in the J plane as determining the scattering amplitude, is the only known procedure that allows such a correlation. The first part of this thesis will be devoted to a study of this photoproduction process using the Regge pole approximation.

The photoproduction process possesses a certain simplicity not to be found in πN , KN , or NN scattering. The Pomeranchuk trajectory which is supposed to dominate πN , KN , and NN scattering will not contribute to $\gamma + N \rightarrow \pi + N$. Furthermore, the rather mysterious trajectory (or cut), the P' , which has been introduced for the sole purpose of supplying another parameter to fit existing data, will also be absent in photoproduction. The theoretical analysis of photoproduction is thereby simplified. In π^+ photoproduction, only the ρ and π trajectories contribute, and at high enough energies, the ρ dominates. This is to be compared with πN , KN , and NN scattering where the ρ trajectory gets lost in much larger P , P' , and ω effects.

In addition to supplying a determination of parameters like $j_\rho(0)$, $j_\pi(0)$,... the photoproduction processes may be used to check other interesting aspects of the theory. For example, it is believed that the contribution of a trajectory, say the ρ , to a process like $\gamma + N \rightarrow \pi + N$ is given by

$$\frac{d\sigma(E,t)}{d\Omega} \rightarrow f_{\rho\pi\gamma}(t) f_{\rho NN}(t) E^{4j_\rho(t)-2}$$

Note that the coefficient of $E^{4j_\rho(t)-2}$ has been factored into two pieces, one depending only on the π - γ - ρ trajectory interaction, the other on the N - N - ρ trajectory interaction. $f_{\rho NN}(t)$ would also appear in $\pi + N \rightarrow \pi + N$ in the form

$$\frac{d\sigma(E,t)}{d\Omega} \rightarrow f_{\rho\pi\pi}(t) f_{\rho NN}(t) E^{4j_\rho(t)-2}$$

Now, by studying the ρ contribution to

$$\gamma + N \rightarrow \pi + N_{3/2^+}^* : \quad \frac{d\sigma}{d\Omega} = f_{\rho\pi\gamma}(t) f_{\rho NN_{3/2^+}^*}(t) E_{\rho}^{4j_{\rho}(t)-2}$$

and

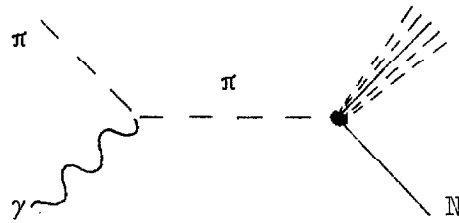
$$\pi + N \rightarrow \pi + N_{3/2^+}^* : \quad \frac{d\sigma}{d\Omega} = f_{\rho\pi\pi}(t) f_{\rho NN_{3/2^+}^*}(t) E_{\rho}^{4j_{\rho}(t)-2}$$

where $N_{3/2^+}^*$ is the $I = 3/2$, $J = 3/2$, parity + πN resonance at 1238 MeV we may test the validity of this factorization. It is interesting to note that only the ρ and π trajectories contribute to the reactions producing $N_{3/2^+}^*$. The P , P' , ω , and ϕ trajectories are not present because they carry zero isotopic spin. The factor theorem would also allow one to relate reactions like $\pi N \rightarrow \pi N$, $\gamma N \rightarrow \pi N$, and $\gamma\pi \rightarrow \pi\pi$. The latter reaction occurs in processes like $\gamma + N \rightarrow \pi + \pi + N$ where a single π is exchanged or in $\pi + \text{Nucleus} \rightarrow \pi + \pi + \text{Nucleus}$ where a virtual γ is exchanged.

From a technical point of view, a theoretical study of photoproduction is interesting because it involves questions of gauge invariance and its effects on the Regge pole theory. As is well known, the π pole in photoproduction appears in the dispersion theory as a consequence of gauge invariance and kinematics (the η which has the same space-time properties as the π does not contribute to photoproduction). It would be interesting to see exactly how the Regge pole formalism gets modified in the case of the π . The factor theorem cannot be correct as it stands for the π trajectory in photoproduction because for $t \approx \mu_{\pi}^2$ nucleon exchange diagrams must

also be considered to preserve gauge invariance.

A study of the pion trajectory at high energies is of interest because of its effect on the Drell process. In 1960 Drell indicated that a great number of strongly interacting particles might be produced with a high energy γ ray beam. The important diagram for the process was single pion exchange:



Because the π pole is so close to the physical region, the one pion exchange contribution is expected to be large despite the presence of an electromagnetic coupling. It is through this mechanism that the Stanford linear electron accelerator will produce beams of strongly interacting particles. As we have previously indicated, the effect of the Regge pole approximation has been a damping of all cross-sections at high energies. It would be very interesting to know exactly how much Drell's original estimate will have to be modified. Because $j_{\pi}(0)$ is not much less than $j_{\pi}(\mu_{\pi}^2)$, it is presently unknown at what energies Regge pole effects become important.

Despite the great uncertainties present in the theory, a study of the photoproduction process does seem justified at this time.

In the second half of this thesis we examine the β -decay of strange particles paying special attention to $K^+ \rightarrow \pi^0 + \mu^+ + \nu$, a process that has been studied to some extent experimentally. This decay mode is governed by the vector current s_{μ} the strange particle

counterpart of the vector current j_μ that appears in neutron β -decay. Recently, convincing evidence^{*} has been obtained verifying the Feynman-Gell-Mann conjecture that $\partial_\mu j_\mu = 0$. The study of $K^+ \rightarrow \pi^0 + \mu^+ + \nu$ is undertaken in an attempt to obtain information about $\partial_\mu s_\mu$. The experimental information available on this decay mode comes from two different groups who seemingly obtain conflicting results. The theory that we propose assumes $\partial_\mu s_\mu$ slowly varying. With this assumption, we find that the data of both groups may be fitted by taking $\partial_\mu s_\mu$ to be small. Since there is always the possibility that one of the experiments is wrong, it would not be fair to conclude that the theory has been experimentally confirmed.

We hope that a knowledge of the divergence $\partial_\mu s_\mu$ will help in reconstructing the current itself, thereby leading to a complete theory for the β -decay of strange particles.

* "Experimental test of the Conserved Vector Current Theory on the Beta Spectra of B^{12} and N^{12} ", Y.K. Lee, L.W. Mo, and C.S. Wu, P.R.L. 10, 253 (1963)

I. INTRODUCTION

The process $\gamma + N \rightarrow \pi + N$ that we wish to consider is shown in Figure 1. Here we call the four-momentum of the incident photon K , the momentum of the outgoing π meson Q , while P_1 and P_2 denote the initial and final nucleon momenta. The kinematic variables s , t , and \bar{s} convenient for this problem are defined by

$$s = -(P_1 + K)^2, \quad t = -(Q - K)^2, \quad \bar{s} = -(P_2 - K)^2$$

In the barycentric system, these quantities become

$$\begin{aligned} s &= (E_1 + k)^2 = (E_2 + \omega)^2 \\ t &= \mu^2 - 2\omega k + 2qk \cos\theta \\ \bar{s} &= M^2 - 2E_2 k - 2qk \cos\theta \end{aligned} \quad (1)$$

where E_1 , k , and E_2 , ω are the initial nucleon, photon, and final nucleon, pion energies; q is the magnitude of the meson three-momentum while $\cos\theta = \vec{q} \cdot \vec{k} / qk$ is the scattering angle. M and μ are the nucleon and pion masses.

We will compute the differential cross section for this reaction at very high C.M. energies (large s) taking into account the ρ , ω , ϕ , and π trajectories in the t -channel and the $N_{1/2^+}^*$, $N_{3/2^+}^*$, $N_{1/2^-}^*$ trajectories* in the \bar{s} channel, as indicated in Figure 2. Gauge invariance will introduce features into our calculation not to be found in π -N or N-N scattering. In the first part of this paper we will write down an expression for the differential cross section paying little attention to rigor. The remainder of the paper will

* The $N_{3/2^+}^*$ is, for example, the trajectory whose lowest mass "particle" is the $I = 3/2$, parity +, πN resonance at 1238 MeV.

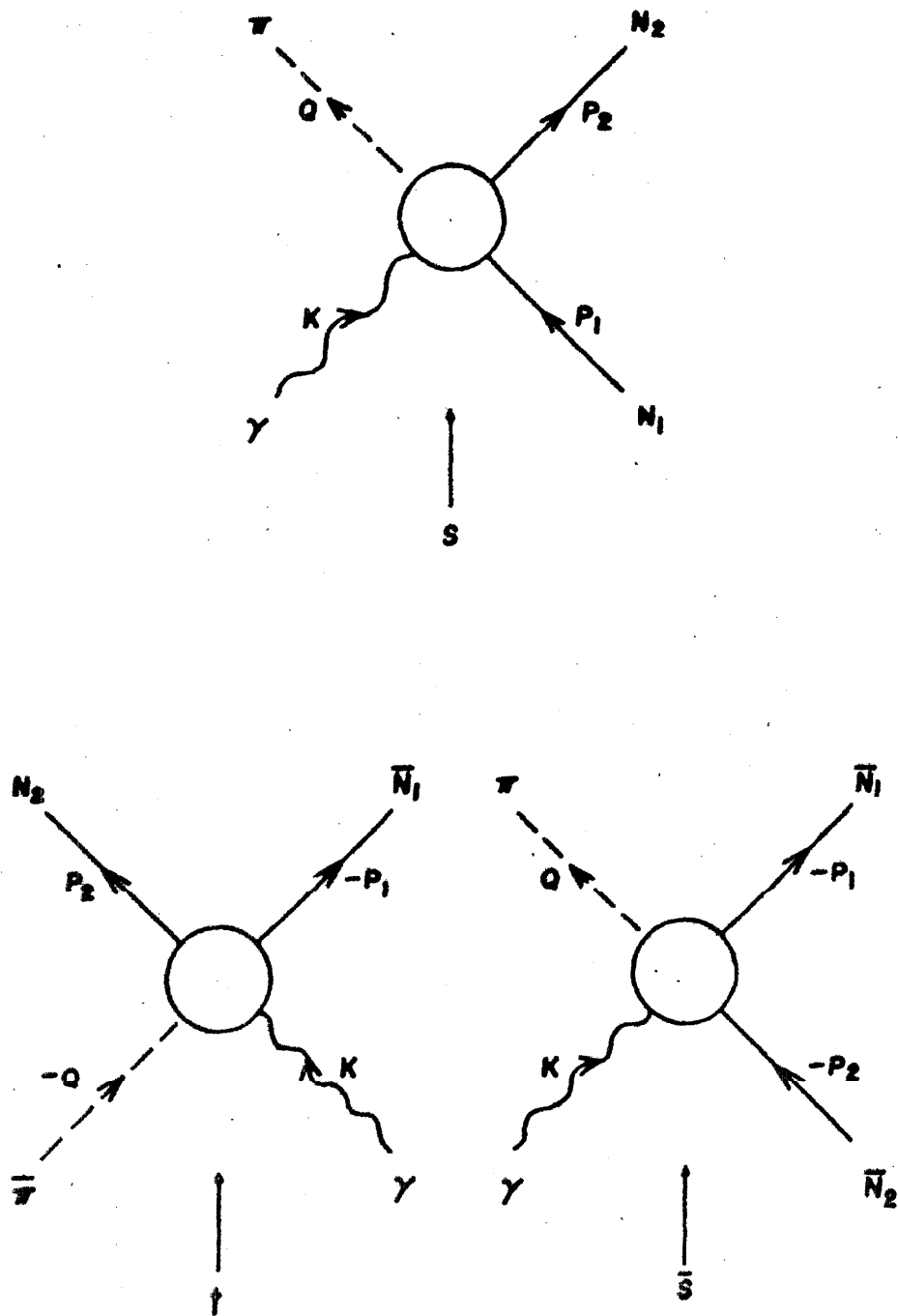


Figure 1: These are the three physical processes that we consider in this paper. They are all described by the same scattering amplitude.

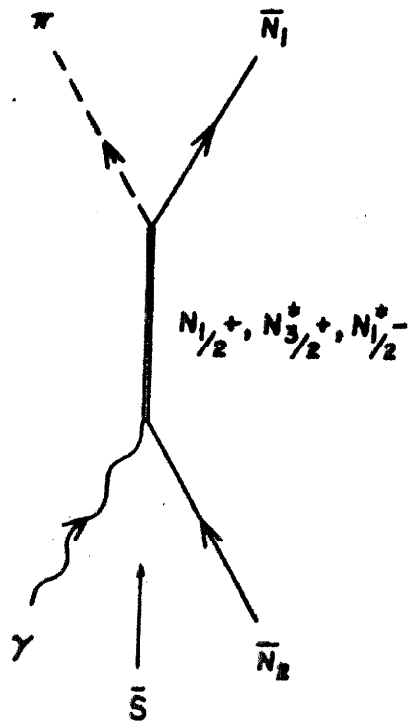
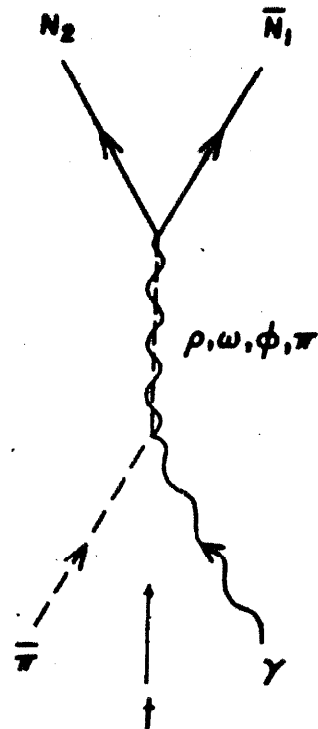


Figure 2: These diagrams indicate the poles or trajectories that we will include in our calculations. Poles in the $t(\bar{s})$ channel determine small angle (large angle) high energy photoproduction.

then be concerned with a verification of this result within the conventional photoproduction formalism.

II. TRAJECTORIES IN THE t -CHANNEL - FORWARD PHOTOPRODUCTION

A. The ω Trajectory

Let us examine the contribution of the ω pole at high energies. If we let the nucleons and the γ ray be in states of definite helicity, then the photoproduction process may be described in terms of $2^3 = 8$ helicity amplitudes. Imposing parity conservation leaves us with only four independent amplitudes which we may take to be ϕ^+ , η^+ where the $+$ superscript refers to the photon helicity, the final state nucleon is always taken to have positive helicity, and ϕ designates nucleon helicity flip, η no helicity flip. Since helicity is not a Lorentz invariant quantity, we must be careful to state what coordinate system we are working in. ϕ and η will always be symbols for the helicity amplitudes in the center-of-mass system.

The $\omega\pi\gamma$ interaction part of the diagram will give a factor $\epsilon_{\nu\alpha\beta\delta} K_\alpha Q_\beta \epsilon_\delta^{(\gamma)}$ to the invariant amplitude T , where $\epsilon_\delta^{(\gamma)}$ is the polarization of the γ . If we go to the $\pi\gamma$ "brick wall frame", defined as that coordinate system where $\vec{q} = -\vec{k}$, and let x be the direction of the incoming γ ray (see Figure 3) then the amplitude will contain the factor

$$\epsilon_{\nu\alpha\beta\delta} K_\alpha Q_\beta \epsilon_\delta^{(\gamma)} = (\epsilon_{\nu x \beta \delta} - \epsilon_{\nu t \beta \delta}) K_\alpha Q_\beta \epsilon_\delta^{(\gamma)} = (-\omega \epsilon_{\nu x t \delta} + k \epsilon_{\nu t x \delta}) K_\alpha \epsilon_\delta^{(\gamma)}$$

Now, $\epsilon_{\nu x t \delta} \epsilon_\delta^{(\gamma)}$ must couple to a four vector, through its ν index, which has to be a linear combination of P_1 and P_2 . If we let

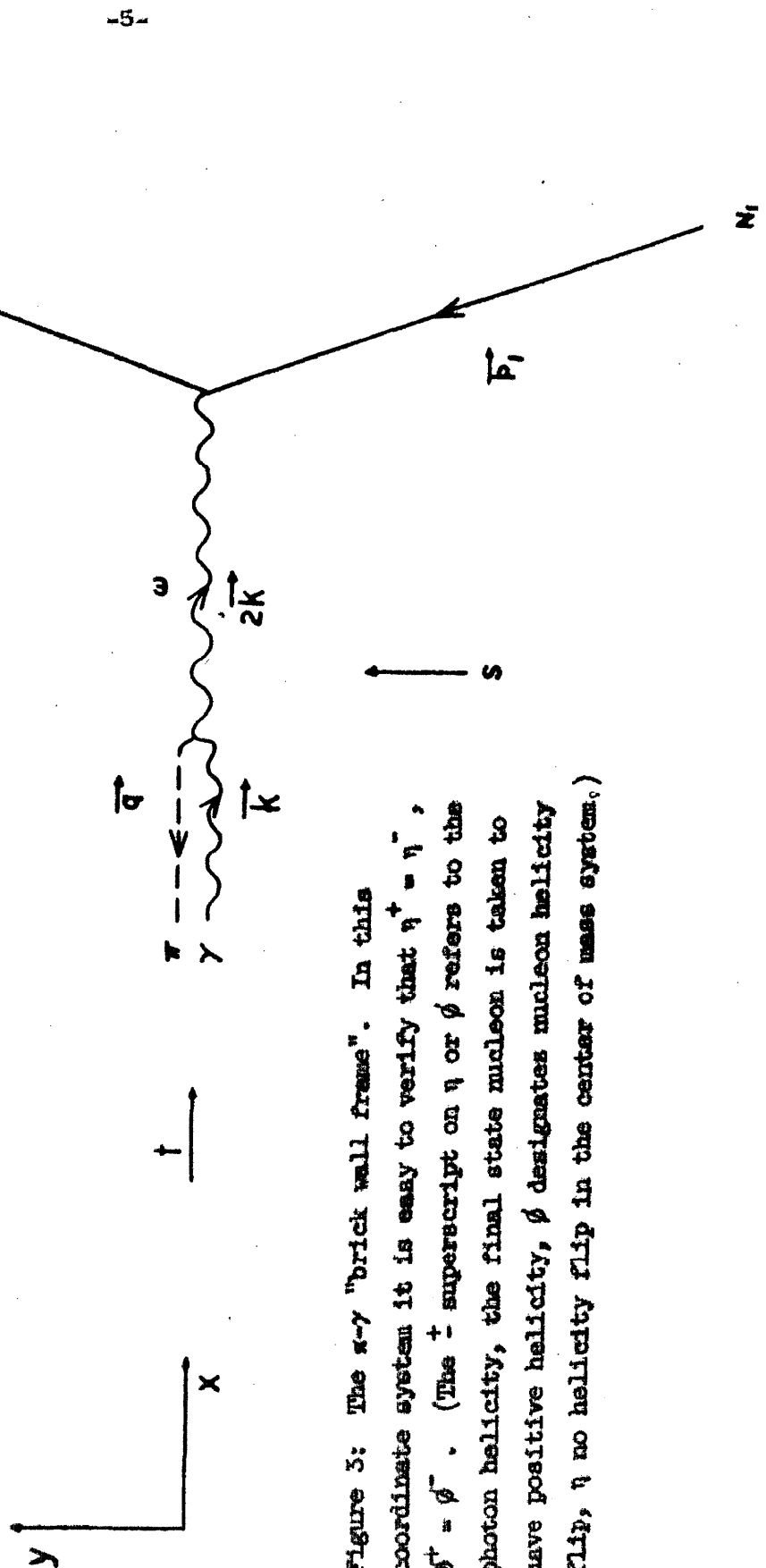


Figure 3: The π - γ "brick wall frame". In this coordinate system it is easy to verify that $\eta^+ = \eta^-$, $\phi^+ = \phi^-$. (The $+$ superscript on η or ϕ refers to the photon helicity, the final state nucleon is taken to have positive helicity, ϕ designates nucleon helicity flip, η no helicity flip in the center of mass system.)

the reaction take place in the xy-plane, then for a non-zero contribution we must have $v = y$, which in turn implies that $\epsilon_8^{(\gamma)}$ will contribute only through its component in the z-direction. Hence,

$$\eta^+ = \eta^- , \quad \phi^+ = \phi^- .$$

In general, we expect both η and ϕ amplitudes to be present. For example, suppose that the helicity of the initial nucleon is positive while that of the final nucleon is negative. Then, in the C.M. system for large s where the mass of the nucleon may be neglected,

$$\frac{1}{2} (1 - \gamma_5) u(\vec{p}_1) = u(\vec{p}_1)$$

$$\bar{u}(\vec{p}_2) \frac{1}{2} (1 - \gamma_5) = \bar{u}(\vec{p}_2)$$

If we look at the ωNN coupling which determines whether we have helicity flip or non-flip we see that couplings of the type

$$\bar{u}(\vec{p}_2) \gamma_\nu u(\vec{p}_1) = \bar{u}(\vec{p}_2) \frac{1}{2} (1 - \gamma_5) \gamma_\nu \frac{1}{2} (1 - \gamma_5) u(\vec{p}_1)$$

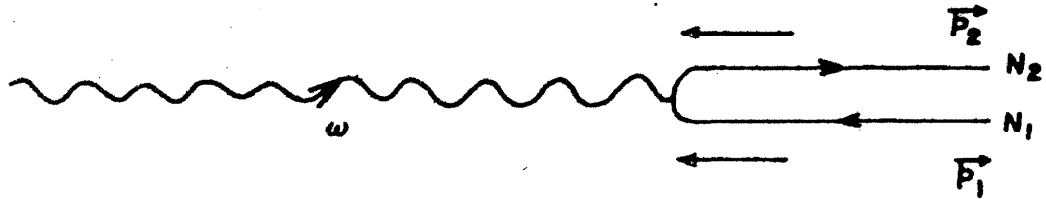
give zero since γ_5 anti-commutes with γ_ν and $(1 - \gamma_5)(1 + \gamma_5) = 0$ ($\frac{1}{2} (1 \pm \gamma_5)$ are projection operators). However,

$$(P_2 - P_1)_\mu \bar{u}(\vec{p}_2) \sigma_{\mu\nu} u(\vec{p}_1)$$

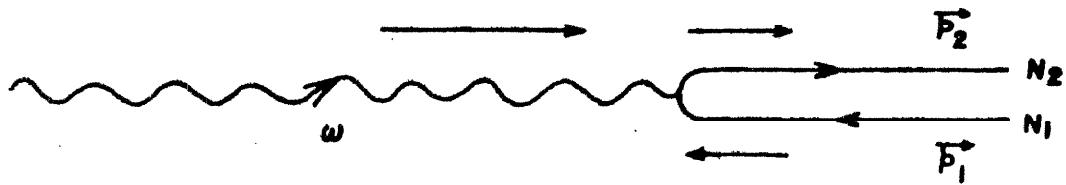
will not equal zero since $\sigma_{\mu\nu}$ contains products of two γ -matrices.

Consequently, the ω pole will give rise to helicity flip ϕ only through its $\sigma_{\mu\nu}$ coupling, and similarly will contribute to no helicity flip η only through the γ_ν term.

Still another way of observing that we must have two independent amplitudes at the ωNN vertex comes by going to the N-N brick wall frame where $\vec{p}_1 = -\vec{p}_2$ (Figure 4). The ω being virtual has both spin zero and spin one pieces. The spin zero part can conserve angular momentum only by interacting with two nucleons that spin in the same



Spin zero piece of virtual ω



Spin one piece of virtual ω

Figure 4: The NN "brick wall frame". In this coordinate system it is apparent that the ω pole gives rise to two independent amplitudes which we have taken to be η and ϕ . The arrows above the momentum lines indicate the spin directions of the particles.

direction and hence contributes only to the spin flip amplitude.

The spin one projection of the ω gives rise only to no helicity flip.

To get the energy dependence of the ω diagram for large s , fixed t , we may for convenience set $\mu = M = 0$. Then, in the π - γ brick wall frame, the kinematic variables become

$$s = - (K + P_1)^2 = - 2 K \cdot P_1 = \sqrt{-t} E_1 - \frac{t}{2}$$

$$t = - (K - Q)^2 = - (2k)^2$$

and for fixed t large s , $p_{1y} \rightarrow E_1 \rightarrow s/\sqrt{-t}$ (see Figure 5). Note that in this coordinate system E_1 goes like s while in the C.M. system E_1 goes like \sqrt{s} .

Consider the ωNN vertex. It will have either the form

$$\bar{u}(\vec{p}_2) \gamma_y u(\vec{p}_1) \quad \text{or} \quad (P_2 - P_1)_\mu \bar{u}(\vec{p}_2) \sigma_{\mu y} u(\vec{p}_1).$$

For fixed t large s ,

$$\bar{u}(\vec{p}_2) \gamma_y u(\vec{p}_1) \sim p_{1y} \rightarrow E_1 \rightarrow s/\sqrt{-t} \quad \text{(no helicity flip)}^*$$

and

$$(P_2 - P_1)_\mu \bar{u}(\vec{p}_2) \sigma_{\mu y} u(\vec{p}_1) = 2k \bar{u}(\vec{p}_2) \sigma_{xy} u(\vec{p}_1) \sim E_1 \rightarrow s/\sqrt{-t} \quad \text{(helicity flip)}$$

Consequently, the invariant amplitude T will go like

$$T \sim (\omega\pi\gamma \text{ vertex}) \times \frac{1}{t - m_\omega^2} \times (\omega NN \text{ vertex}) \sim s$$

for both flip and non-flip. T is related to the helicity amplitudes through a phase space factor. This gives rise to the relations

$$\left\{ \begin{array}{l} \eta \\ \phi \end{array} \right\} \sim \frac{T}{\sqrt{s}}.$$

* The symbol " \sim " means "goes in its s dependence like"; " \rightarrow " means "which approaches".

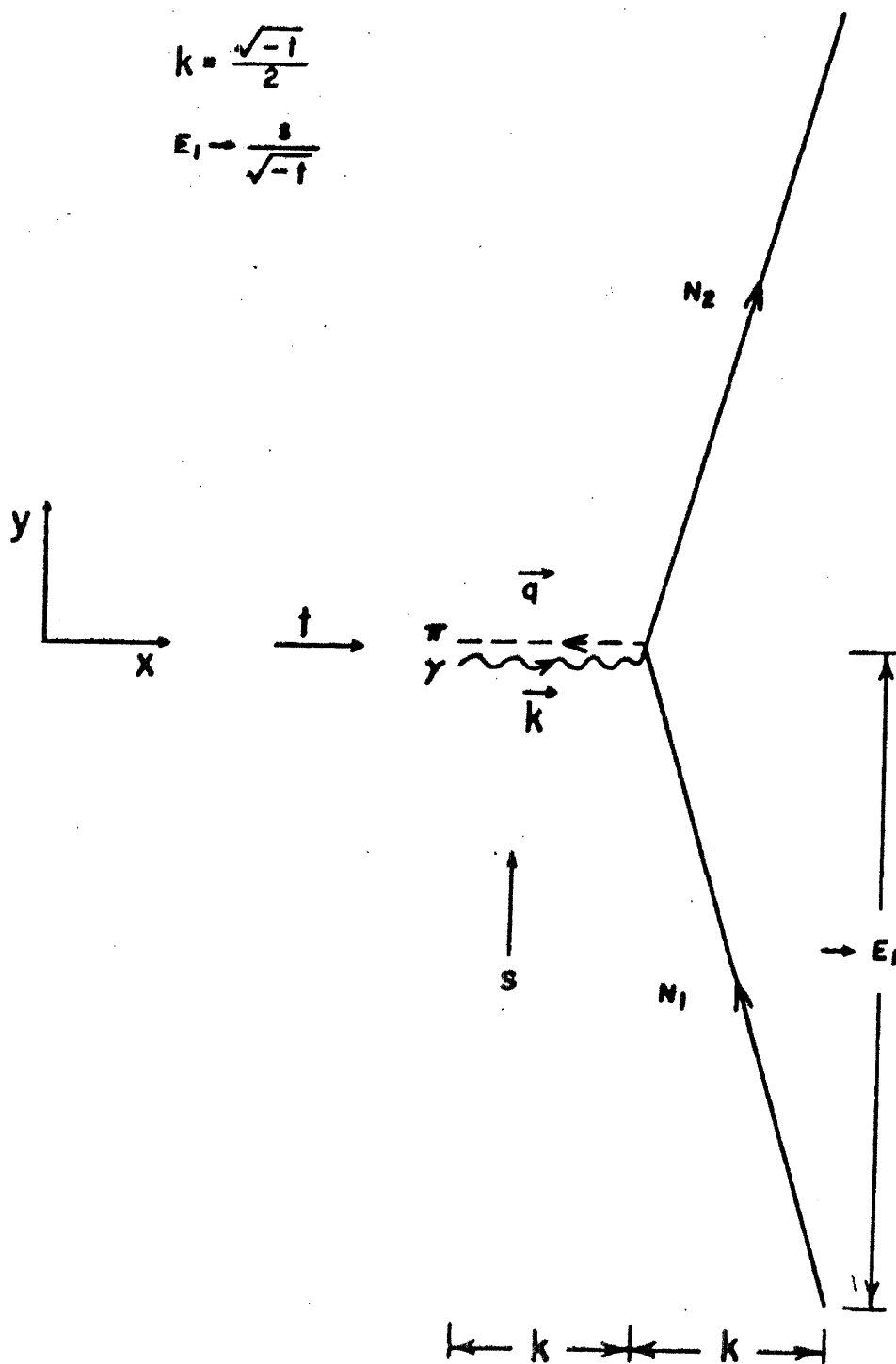


Figure 5: The π - γ "brick wall frame". We compute the s dependence of the ω trajectory contribution to photoproduction. The masses μ and M have been set equal to zero for convenience.

The differential cross section at fixed t and large s is given by

$$\frac{d\sigma_{\omega}}{d\Omega} \rightarrow |\phi|^2 + |\eta|^2 \sim s = s^{2j_{\omega}-1}$$

where j_{ω} is the spin of the ω .

Suppose that we exchange, instead of an ω , another (hypothetical) particle on the ω trajectory, a spin-3 ω called the ω^* . This particle will couple at each vertex with 2 more indices than the ω , indices which must be saturated by the four momenta of the external particles.* This means that two more powers of s will appear at the ω^*NN vertex with the result that $d\sigma_{\omega^*}/d\Omega \sim s^5 = s^{2j_{\omega^*}-1}$. In general then, $d\sigma/d\Omega \sim s^{2j-1}$ or equivalently,

$$d\sigma/dt \sim s^{2j-2}$$

at large s fixed t for any particle of spin " j " on the ω trajectory.

If we make the Regge pole approximation where we treat the contribution of the entire trajectory as one term, rather than using individual contributions from each particle that lies on the trajectory, we have

$$\frac{d\sigma_{\omega}^{+}}{dt} = \frac{1}{4\pi} (|\eta_{\omega}^{+}(t)|^2 + |\phi_{\omega}^{+}(t)|^2) |\zeta_{\omega}(t)|^2 \left(\frac{s}{s_{\omega}}\right)^{2j_{\omega}(t)-2} \quad (2)$$

where $\zeta_{\omega}(t)$ is the signature factor,

* The number of indices necessary to describe the field of an integral spin particle is just equal to the spin of the particle.

$$\xi_{\omega}(t) = \frac{1 - e^{-\pi j_{\omega}(t)}}{2 \sin \pi j_{\omega}(t)},$$

$$\eta_{\omega}^{+}(t) = \eta_{\omega}^{-}(t), \quad \phi_{\omega}^{+}(t) = \phi_{\omega}^{-}(t),$$

and s_{ω} is any convenient (mass)² making s/s_{ω} dimensionless. In terms of the invariant T matrix,

$$T_{\omega}^{\pm} = \xi_{\omega}(2 s_{\omega}) (s/s_{\omega})^{j_{\omega}(t)} \times (\eta_{\omega}^{\pm}(t) \text{ or } \phi_{\omega}^{\pm}(t)).^{*}$$

Remember that this is the differential cross section for the photo-production process leading to final state nucleons of positive helicity. With the help of parity conservation, averaging over initial and summing over final helicity states:

$$\left\langle \frac{d\sigma_{\omega}}{dt} \right\rangle = \frac{1}{2} \left(\frac{d\sigma_{\omega}^{+}}{dt} + \frac{d\sigma_{\omega}^{-}}{dt} \right) = \frac{d\sigma_{\omega}^{\pm}}{dt}$$

If we now put in the trivial isotopic spin analysis, we see that the $I = 0$ ω contributes to physically observed processes with weights:

$\gamma + p \rightarrow \pi^0 + p$	$\gamma + n \rightarrow \pi^0 + n$	$\gamma + p \rightarrow \pi^{+} + n$	$\gamma + n \rightarrow \pi^{-} + p$
1	1	0	0

* When working with the general photoproduction formalism, η^{\pm} or ϕ^{\pm} will refer to the non-Lorentz invariant helicity amplitudes (where $d\sigma^{\pm}/d\Omega = q/k \{ |\eta^{\pm}|^2 + |\phi^{\pm}|^2 \}$). When dealing with the Regge pole formalism, $\eta^{\pm}(t)$ or $\phi^{\pm}(t)$ will refer to the Lorentz invariant amplitudes defined above.

B. The ϕ and ρ Trajectories

Because the space-time and isospin structure of the ϕ and ω are identical, the ϕ pole or trajectory contribution to photoproduction is just that given in the section above with ϕ replacing ω throughout. Similarly, the section above is also valid for the ρ trajectory, but because of isotopic spin differences, the weights to be used are:

$$\begin{array}{cccc} \gamma + p \rightarrow \pi^0 + p & \gamma + n \rightarrow \pi^0 + n & \gamma + p \rightarrow \pi^+ + n & \gamma + n \rightarrow \pi^- + p \\ 1 & -1 & \sqrt{2} & \sqrt{2} \end{array}$$

C. The π Trajectory

In order to make a gauge invariant calculation, we must consider simultaneously the three graphs of Figure 6 with their invariant amplitudes. Since we will be interested in the region of t near μ^2 , it may superficially appear that only the pion pole diagram is important. However, in the $\gamma - \pi$ brick wall frame ($\vec{q} = -\vec{k}$), using the Coulomb gauge, the factor $\epsilon_{\mu}^{(\gamma)} q_{\mu}$ that multiplies the pion pole gives $\epsilon_{\mu}^{(\gamma)} q_{\mu} = \vec{\epsilon} \cdot \vec{q} = -\vec{\epsilon} \cdot \vec{k} = 0$ so that the π pole graph completely vanishes, leaving the two nucleon diagrams to conspire to give the well known $1/(t-\mu^2)$ photoproduction retardation term. In this gauge and coordinate system, it is easy to see that the $1/(t-\mu^2)$ factor that will eventually appear in the amplitude has little to do with the pion pole diagram, but is rather an effect due to gauge invariance and kinematics, the μ^2 in the $1/(t-\mu^2)$ pole referring to the mass of the external π .

At high energies in the C.M. system, all graphs lead only to nucleon helicity flip ($\eta^{\pm} = 0$). For example, in the pion pole diagram,

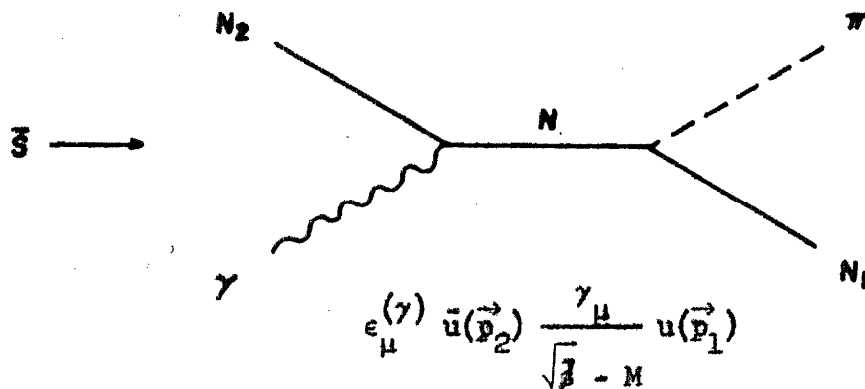
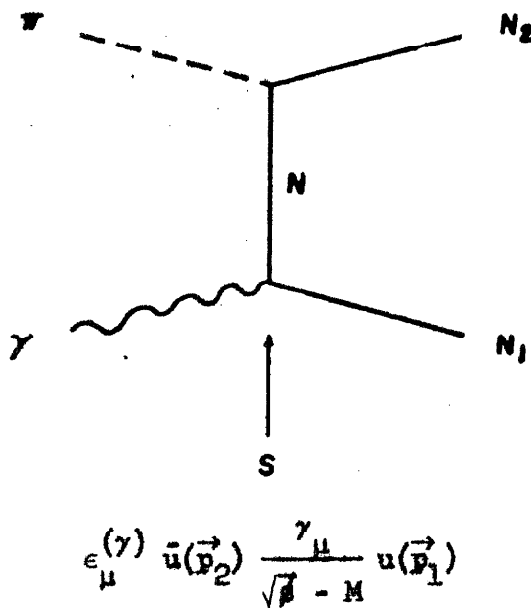
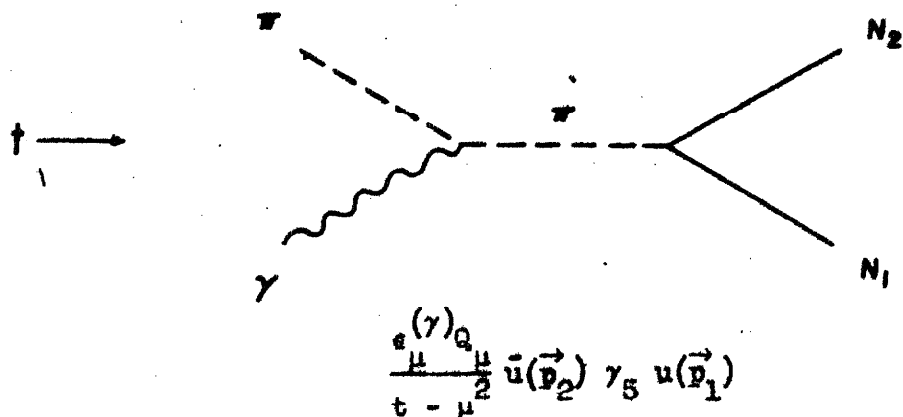


Figure 6: In order to make a gauge invariant calculation of the " π pole" contribution to photoproduction, we consider simultaneously these three, Feynman graphs with their indicated amplitudes.

the no helicity flip amplitude is proportional to

$$\bar{u}(\vec{p}_2) \gamma_5 u(\vec{p}_1) = \bar{u}(\vec{p}_1) \frac{(1+\gamma_5)}{2} \gamma_5 \frac{(1+\gamma_5)}{2} u(\vec{p}_1) = 0. *$$

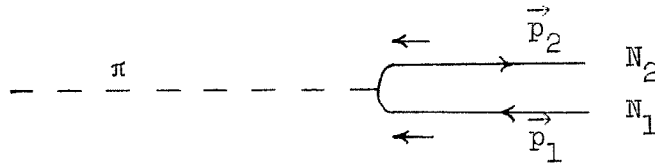
Similarly, since the virtual nucleons in the other two graphs will possess large energies, there must be helicity flip at the πNN vertex and no helicity flip at the γNN vertex, that is, net helicity flip. **

Looking for a moment at the nucleon pole in the s-channel, going to the C.M. system, and using the Coulomb gauge, we note that

$$\epsilon_\mu^{(\gamma)} \bar{u}(\vec{p}_2) \frac{\gamma_\mu}{\sqrt{s} - M} u(\vec{p}_1) = \vec{\epsilon} \cdot \left[a(s,t) \vec{p}_1 + b(s,t) \vec{p}_2 \right] = b(s,t) \vec{\epsilon} \cdot \vec{p}_2 = 0$$

unless $\vec{\epsilon}$ is in the plane of production. Exactly the same result holds for the other two diagrams yielding $\phi^+ = -\phi^-$.

* It is amusing to note that in the N-N brick wall frame where nucleon energies are not necessarily large, there still can only be helicity flip from the π pole.



Since the π has spin zero incoming and outgoing nucleons must spin in the same direction, hence helicity flip.

** This argument works only when the energy of the virtual nucleon is large. For large s fixed \bar{s} the nucleon pole in the \bar{s} -channel gives both flip and no-flip.

The energy dependence of the diagrams may now be trivially determined. The two nucleon graphs approach a constant at fixed t large s ($\bar{s} = -s - t + 2M^2 + \mu^2$). The pion pole term looks like it goes as $Q_\mu \sim \sqrt{s}$, but fixed t large s means nearly forward scattering ($\theta \sim 2\sqrt{\frac{-t}{s}}$). Therefore the polarization $\vec{\epsilon}$, being perpendicular to the incident γ , is now almost perpendicular to the final pion direction. Explicitly,

$$\epsilon_\mu^{(\gamma)} Q_\mu = q \cos(\vec{\epsilon}, \vec{q}) \sim \sqrt{s} \sqrt{\frac{-t}{s}} = \sqrt{-t},$$

so that this diagram also becomes energy independent. The requirement that the γ ray have only two directions or polarization has effectively reduced the s dependence of the amplitude. The helicity amplitudes go like $1/\sqrt{s}$ and $d\sigma/d\Omega \sim 1/s \sim s^{2j_\pi - 1}$.

If instead of a pion we were to exchange another (hypothetical) particle π^* on the pion trajectory, we would no longer need the nucleon graphs to maintain gauge invariance. The π^* pole that would now appear in the amplitude would no longer be of kinematic origin and would contribute to the differential cross section with s dependence $d\sigma/d\Omega \sim s^{2j_{\pi^*} - 1}$, $d\sigma/dt \sim s^{2j_{\pi^*} - 2}$.

In a gauge invariant but otherwise conventional Regge pole treatment of the pion trajectory, we find

$$\eta_\pi^\pm(t) = 0, \quad \phi_\pi^+(t) = -\phi_\pi^-(t), \quad \frac{d\sigma_\pi^\pm}{dt} = \frac{1}{4\pi} \frac{|\phi_\pi^+(t) j_\pi(t) \zeta_\pi(t)|^2}{(t - \mu^2)^2} \left(\frac{s}{s_\pi}\right)^{2j_\pi(t) - 2} \quad (3)$$

where

$$\zeta_\pi(t) = \frac{1 + e^{-i\pi j_\pi(t)}}{2\sin\pi j_\pi(t)}$$

$j_\pi(\mu^2) = 0$, and s_π is some constant making s/s_π dimensionless. The

$1/(t-\mu^2)$ comes from the kinematics, the μ^2 referring to the mass of the external π . For $t \approx \mu^2$, the pole that comes from $1/\sin \pi j_\pi(t)$ is cancelled by the $j_\pi(t)$ factor in the numerator leaving us with a kinematic singularity in the cross section. For t such that $j_\pi(t) = 2, 4, \dots$ the cross section receives a pole in the usual manner through the $1/\sin \pi j_\pi(t)$ term. Finally,

$$\left\langle \frac{d\sigma_\pi}{dt} \right\rangle = \frac{1}{2} \left(\frac{d\sigma_\pi^+}{dt} + \frac{d\sigma_\pi^-}{dt} \right) = \frac{d\sigma_\pi^\pm}{dt}.$$

The π trajectory contributes to physically observed processes with weights:

$$\begin{array}{cccc} \gamma + p \rightarrow \pi^0 + p & \gamma + n \rightarrow \pi^0 + n & \gamma + p \rightarrow \pi^+ + n & \gamma + n \rightarrow \pi^- + p \\ 0 & 0 & \sqrt{2} & -\sqrt{2} \end{array}$$

Combining the ω , ρ and π trajectories, we may write

$$\frac{d\sigma_\pi^\pm}{dt} = \frac{1}{4\pi} \sum_{i,k=\omega,\rho,\pi} \text{Re } \zeta_i^* \zeta_k \left(\frac{s}{s_i}\right)^{j_i-1} \left(\frac{s}{s_k}\right)^{j_k-1} (\eta_i^+ \eta_k^+ + \phi_i^+ \phi_k^+) \quad (4)$$

where: (a) the amplitudes η_i, ϕ_i are presumably real for physical values of t in the s -channel; (b) ϕ_π^\pm stands for $\phi_\pi^\pm j_\pi/(t-\mu^2)$; (c) $d\sigma_\pi^\pm/dt$ is the cross section for scattering into a final state π and positive

helicity nucleon. Parity conservation gives the negative helicity case.

For an unpolarized γ -ray beam and target,

we have $\frac{d\sigma}{dt} = \frac{1}{2} \left(\frac{d\sigma^+}{dt} + \frac{d\sigma^-}{dt} \right)$. Note that $d\sigma^+/dt$ is no longer equal to $d\sigma^-/dt$ in the reactions $\gamma + p \rightarrow \pi^+ + n$ and $\gamma + n \rightarrow \pi^- + p$ because while $\eta_\rho^+ = \eta_\rho^-$ and $\phi_\rho^+ = \phi_\rho^-$, we also have $\phi_\pi^+ = -\phi_\pi^-$. The difference $\frac{d\sigma^+}{dt} - \frac{d\sigma^-}{dt}$ is therefore just a measure of the interference between the pion trajectory and the helicity flip parts of the ρ trajectory.

As has been indicated by Gell-Mann⁽¹⁾ and Gribov and Pomeranchuk⁽²⁾, it may not be unreasonable to suppose that $\eta(t)$ and $\phi(t)$ can be factored and looked upon as the product of two t -dependent couplings of the Regge pole to the NN and $\pi\gamma^*$. Then, we may write

$$\eta_{\frac{1}{2}}^\pm(t) = \eta_{\frac{1}{2}}^i(t) \gamma_{\frac{1}{2}}^\pm(t) \quad , \quad \phi_{\frac{1}{2}}^\pm(t) = \phi_{\frac{1}{2}}^i(t) \gamma_{\frac{1}{2}}^\pm(t). \quad (5)$$

$\eta_{\frac{1}{2}}^i$ and $\phi_{\frac{1}{2}}^i$ may be determined independently from π -N or N-N scattering. With the conventions we have chosen, $\eta_{\frac{1}{2}}^i$ and $\phi_{\frac{1}{2}}^i$ are identical to Wagner's⁽³⁾ $\eta_{\frac{1}{2}}^i$ and $\phi_{\frac{1}{2}}^i$.

D. Polarization of the Recoil Nucleon

Let $\eta_{\frac{1}{2}}^\pm$ and $\phi_{\frac{1}{2}}^\pm$ be helicity amplitudes with the final nucleon having negative instead of the customary positive helicity. Also, \vec{p}_2 of the final nucleon and let y_2 lie along let z_2 be in the direction $\vec{p}_1 \times \vec{p}_2$, perpendicular to the production plane. Then, if "a" and "a" are the amplitudes for the recoil nucleon to have + and - helicity, the expected value of the spin in the y_2 direction $\langle s_{y_2} \rangle$ is just $\langle s_{y_2} \rangle = \text{Im } a^* \underline{a}$. From this immediately follows, for example,

* Because of gauge invariance, there are additional complications for the π trajectory.

$$\langle s_{y_2} \rangle^{1,1/2} = \sum_{i,k=\omega,\phi,\rho,\pi} \frac{\text{Im} \left(\frac{\eta_i^{+*}}{\sqrt{\frac{d\sigma}{d\Omega}}} \frac{\phi_k^+}{\sqrt{\frac{k d\sigma}{g d\Omega}}} \right)}{1,1/2}$$

$$\frac{d\sigma}{d\Omega}^{1,1/2} = \frac{g}{k} \sum_{i,k=\omega,\phi,\rho,\pi} (\text{Re } \eta_i^{+*} \eta_k^+ + \text{Re } \phi_i^{+*} \phi_k^+)$$

where the superscript 1, 1/2 indicates that we are dealing with initial states of photon and nucleon helicities of 1 and 1/2 . Parity conservation yields $\eta_i^+ = -\eta_i^{\bar{+}}$, $\phi_i^+ = \phi_i^{\bar{+}*}$. Using this result and working out the four possible initial helicity combinations, we find

$$\langle s_{y_2} \rangle^{1,1/2} = \langle s_{y_2} \rangle^{-1,-1/2} = \frac{\sum_{i,k} \text{Im } \eta_i^{+*} \phi_k^-}{\frac{k}{g} \frac{d\sigma}{d\Omega}^{1,1/2}} \quad (6)$$

$$\langle s_{y_2} \rangle^{-1,1/2} = \langle s_{y_2} \rangle^{1,-1/2} = \frac{\sum_{i,k} \text{Im } \eta_i^{-*} \phi_k^+}{\frac{k}{g} \frac{d\sigma}{d\Omega}^{-1,1/2}}$$

where

$$\frac{d\sigma}{d\Omega}^{1,1/2} = \frac{d\sigma}{d\Omega}^{-1,-1/2} = \frac{g}{k} \sum_{i,k} (\text{Re } \eta_i^{+*} \eta_k^+ + \text{Re } \phi_i^{-*} \phi_k^-) = \frac{d\sigma}{d\Omega}^+$$

* For phase conventions, see: M. Jacob and G.C. Wick, Ann. Phys. 7, 404 (1959)

$$\frac{d\sigma}{d\Omega}^{-1,1/2} = \frac{d\sigma}{d\Omega}^{1,-1/2} = \frac{g}{4\pi} \sum_{i,k} (\text{Re } \eta_i^{-*} \eta_k^- + \text{Re } \phi_i^{+*} \phi_k^+) = \frac{d\sigma}{d\Omega}^-$$

Polarization of the recoil nucleon resulting from unpolarized initial particles is then trivially

$$\langle s_{y_2} \rangle = \frac{1}{2} \left[\langle s_{y_2} \rangle^{1,1/2} + \langle s_{y_2} \rangle^{-1,1/2} \right]$$

In terms of the Regge pole parameters,

$$\langle s_{y_2} \rangle^{1,1/2} \frac{d\sigma}{dt}^{1,1/2} = \frac{1}{4\pi} \sum_{i,k} \text{Im } \zeta_i^* \zeta_k \left(\frac{s}{s_i} \right)^{j_i-1} \left(\frac{s}{s_k} \right)^{j_k-1} \eta_i^+ \phi_k^- \quad (7)$$

$$\langle s_{y_2} \rangle^{-1,1/2} \frac{d\sigma}{dt}^{-1,1/2} = \frac{1}{4\pi} \sum_{i,k} \text{Im } \zeta_i^* \zeta_k \left(\frac{s}{s_i} \right)^{j_i-1} \left(\frac{s}{s_k} \right)^{j_k-1} \eta_i^- \phi_k^+$$

where $\frac{d\sigma}{dt}^{+1,1/2} \equiv \frac{d\sigma}{dt}^+$ has been given in equation 4.

III. TRAJECTORIES IN THE \bar{s} CHANNEL - BACKWARD PHOTOPRODUCTION

The \bar{s} and t channels are qualitatively different in many respects.

If, for example, we look at the nucleon pole exchange diagram for large s fixed \bar{s} , as indicated in Figure 7, we see that even though an incoming nucleon N_1 of definite helicity, say $+1/2$, couples only to $(1 + \gamma_5)N$, $(1 + \gamma_5)N$ will not be a helicity eigenstate of N (since \bar{s} need not be large compared to M^2). Consequently, when $(1 + \gamma_5)N$ propagates to the $N_2 N_\gamma$ vertex it arrives with both components $(1 + \gamma_5)N$ and $(1 - \gamma_5)N$, giving rise to helicity flip and no flip, regardless of whether the γ -ray is coupled to the charge or the anomalous magnetic moment of the nucleon. This situation is very much more complicated than a consideration of the same diagram for large s fixed t -- see Section II.C, "The π Trajectory". There, \bar{s} was much greater than

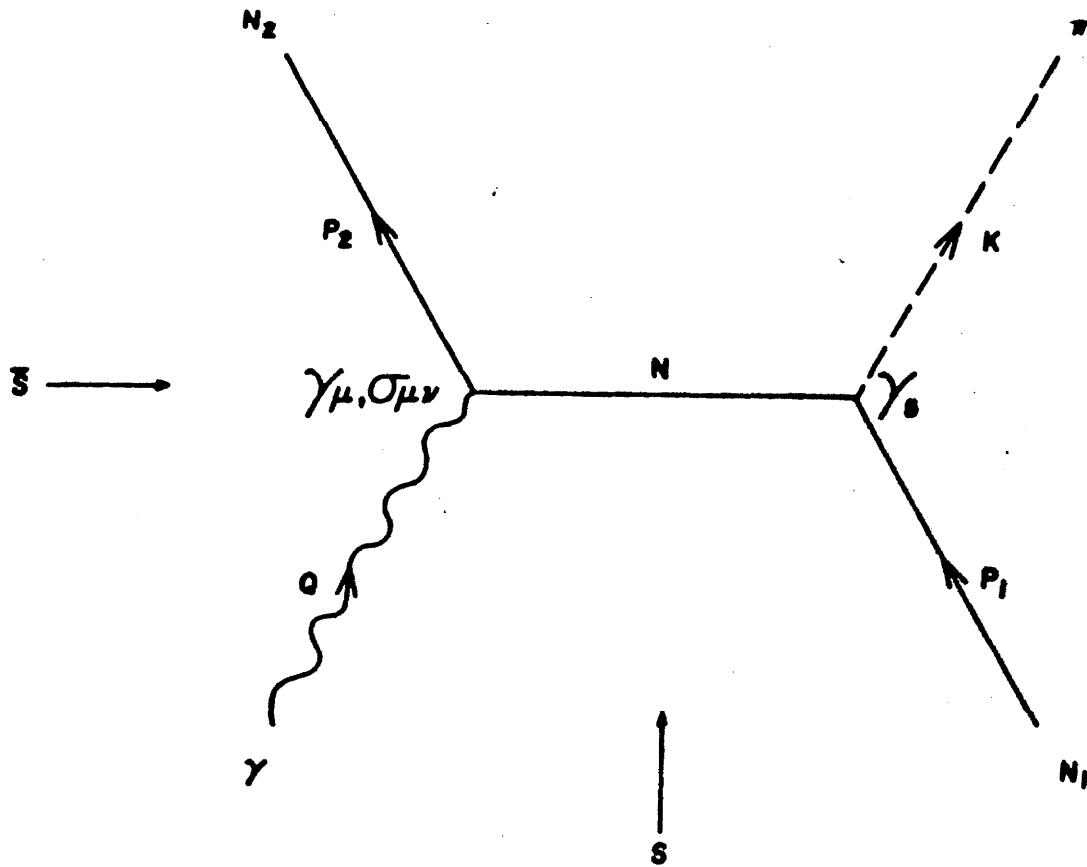


Figure 7: The nucleon pole in the \bar{s} channel. Because the virtual nucleon energy is not necessarily large, simple relationships between the helicity amplitudes do not exist.

M^2 so that $(1 + \gamma_5)N$ represented a definite helicity state of N which remained fixed as N propagated to the $N_2 N_1$ vertex.

Another difference between the \bar{s} and t channels is to be found in a peculiar symmetry. Consider for the moment the process $\gamma + N \rightarrow \pi + N$ described by the S matrix element

$$\langle \vec{p}_2 E_2 h_2; \vec{q} \omega | S | \vec{k} k h; \vec{p}_1 E_1 h_1 \rangle$$

where h, h_1, h_2 are the signs of the helicities. Viewing this reaction as it proceeds backwards in time, we write

$$\begin{aligned} \langle \vec{p}_2 E_2 h_2; \vec{q} \omega | S | \vec{k} k h; \vec{p}_1 E_1 h_1 \rangle &= \\ \langle -\vec{p}_1 -E_1 h_1; -\vec{k} -k h | S | -\vec{q} -\omega; -\vec{p}_2 -E_2 h_2 \rangle \end{aligned}$$

Using invariance under strong (or Schwinger) time reversal T_s and charge conjugation we obtain

$$\begin{aligned} \langle \vec{p}_2 E_2 h_2; \vec{q} \omega | S | \vec{k} k h; \vec{p}_1 E_1 h_1 \rangle &= \\ h_1 h_2 \langle \vec{p}_2 -E_2 h_2; \vec{q} -\omega | S | \vec{k} -k h; \vec{p}_1 -E_1 h_1 \rangle \end{aligned}$$

where, for example,

$$T_s | -\vec{p}_2 -E_2 h_2 \text{ particle} \rangle = -h_2 \lambda^* \langle \vec{p}_2 -E_2 h_2 \text{ antiparticle} | ;$$

λ^* is a phase factor taken such that $\lambda^* \lambda = 1$. In terms of our previous notation,

$$\eta^\pm(-\sqrt{s}) = \eta^\pm(\sqrt{s}), \quad \phi^\pm(-\sqrt{s}) = -\phi^\pm(\sqrt{s}). \quad (8)$$

Since the s and \bar{s} channels describe essentially the same reaction (Figure 1), any symmetry present in the variable \sqrt{s} will have its counterpart in $\sqrt{\bar{s}}$. This symmetry in the $\sqrt{\bar{s}}$ variable will be made more explicit below.

A. The Nucleon Trajectory

Because of gauge invariance we must consider the same diagrams

that were given in Figure 6, but this time for large s fixed \bar{s} .

The nucleon pole in the \bar{s} channel will dominate in the C.M. system giving an amplitude T that goes like $s^{1/2} = s^{j_N}$. For a general "particle" of spin j on the nucleon trajectory, $T \sim s^j$, $\frac{d\sigma}{dt} \sim s^{2j-2}$.

Unfortunately, the remaining properties of this trajectory are rather involved. We shall only state the results, leaving the verification to the appendix.

The helicity amplitudes may be written as

$$\begin{aligned}\eta^- &\rightarrow \frac{-i\sqrt{2}}{16\pi} \bar{s} \left[\frac{\mu^2 M}{s} A_2 + A_3 - A_4 \right] \\ \eta^+ &\rightarrow \frac{-i\sqrt{2}}{8\pi} \left[M A_1 - (M^2 + \bar{s}) A_4 + \frac{s}{2} (A_3 + A_4) \right] - \eta^- \\ \phi^+ &\rightarrow \frac{i\sqrt{2}\sqrt{-\bar{s}}}{16\pi} \left[s A_2 + M (A_3 - A_4) \right] \\ \phi^- &\rightarrow \frac{i\sqrt{2}\sqrt{-\bar{s}}}{8\pi} \left[A_1 - 2M A_4 + M (A_3 + A_4) \right] - \phi^+\end{aligned}\tag{9}$$

where the A_i 's are scalar functions of s , t , \bar{s} .^{*} The three possible isotopic spin indices have been momentarily suppressed. The A_i 's are convenient to work with because they obey simple crossing relations.

In the Regge pole approximation,

$$\begin{aligned}A_1 &= \frac{1}{2} \left[\bar{W}_- \zeta_+ \left(\frac{s}{s_N} \right)^{\alpha_+} a_+ + \bar{W}_+ \zeta_- \left(\frac{s}{s_N} \right)^{\alpha_-} a_- + M \zeta_+ \left(\frac{s}{s_N} \right)^{\alpha_+} b_+ + M \zeta_- \left(\frac{s}{s_N} \right)^{\alpha_-} b_- \right] \\ A_2 &= \frac{1}{2} \left[\bar{W}_+ \zeta_+ \left(\frac{s}{s_N} \right)^{\alpha_+ - 1} \frac{b_+}{s_N} + \bar{W}_- \zeta_- \left(\frac{s}{s_N} \right)^{\alpha_- - 1} \frac{b_-}{s_N} \right]\end{aligned}\tag{10}$$

* The A_i 's are identical to those of: James Stutsman Ball, Phys. Rev. 124, 2014 (1961). Also, $A_{1,2,3,4} = A,B,C,D$ of CGLN: G. F. Chew, M. L. Goldberger, F. E. Low, Y. Nambu, Phys. Rev. 106, 1345 (1957). See Appendix I for the definition of A_i ($A_i^{(+)(-)(0)}$).

$$A_3 = \frac{1}{2} \left[\zeta_+ \left(\frac{s}{s_N} \right)^{\alpha_+} a_+ + \zeta_- \left(\frac{s}{s_N} \right)^{\alpha_-} a_- + \zeta_+ \left(\frac{s}{s_N} \right)^{\alpha_+} \frac{b_+}{2} + \zeta_- \left(\frac{s}{s_N} \right)^{\alpha_-} \frac{b_-}{2} \right] \quad (10)$$

$$A_4 = -A_3 \pm (\bar{s} - M^2) \left[\zeta_+ \left(\frac{s}{s_N} \right)^{\alpha_+ - 1} \frac{b_+}{s_N} + \zeta_- \left(\frac{s}{s_N} \right)^{\alpha_- - 1} \frac{b_-}{s_N} \right]$$

where the \pm subscript on the functions indicates that the variable is $\pm\sqrt{s}$, e.g.,

$$a_{\pm} = a_N(\pm\sqrt{s}); \quad \alpha_{\pm} = j_N(\pm\sqrt{s}) - 1/2,$$

$$\zeta_{\pm} \text{ is the nucleon signature factor equal to } \frac{1+e^{-i\pi\alpha_{\pm}}}{2 \sin \pi\alpha_{\pm}},$$

$$\bar{W}_{\pm} = \pm\sqrt{s} + M,$$

and s_N is some convenient (mass)². The \pm sign multiplying all expressions in brackets relates to isotopic spin and crossing symmetry.

The top sign is to be taken for the amplitudes $A_i^{(+)}, (0)$, the bottom for $A_i^{(-)}$, where $(+), (0), (-)$ are isotopic spin indices.

Note that the A_i are even under the interchange $\sqrt{s} \leftrightarrow -\sqrt{s}$ as expected. The cross section $\frac{d\sigma_N}{ds}$ goes like

$$\frac{d\sigma_N}{ds} \sim \left(\frac{s}{s_N} \right)^{2j_N(\pm\sqrt{s}) - 2}$$

B. The $N_{3/2}^+$ and $N_{1/2}^-$ Trajectories

Because the $N_{1/2}^*$ has parity opposite that of the nucleon, but one unit more of spin angular momentum, the results for the N trajectory may be carried over directly to the $N_{1/2}^*$ case, changing the signature from + to - and replacing N everywhere by $N_{1/2}^*$.

To obtain the results for the $N_{3/2}^{*+}$ from the N, again we must change the signature from + to -, and because the parities are the same while the spin differs by one unit, we must exchange the subscripts \pm on ζ , α , a, and b. The subscript on \bar{W} remains the same. Then, we replace N by $N_{3/2}^{*+}$ throughout. Finally, differences in isotopic spin must be taken into account. For details, see Appendix IV. A,B.

Kycia and Riley⁽⁴⁾ have recently indicated the possible existence of another Regge trajectory $N_{3/2}^{*-}$ that would be responsible for a "shoulder" and bump in π -N scattering at total center-of-mass energies of 1650 and 2360 MeV. If this trajectory does exist, it would contribute in a form identical to that of the $N_{3/2}^{*+}$, but with opposite signature.

IV. Conclusions

For the trajectories considered in this paper, the real part of $j(t)$ or $j(\bar{s})$ has been plotted as a function of t or \bar{s} in Figures 8 and 9. The straight line approximations to the trajectories should not be taken too seriously. They will only be used for order of magnitude calculations. More accurate results will have to await a determination of the trajectories from πN , $K\pi$, NN . . . scattering experiments.

Only the π and ρ trajectories contribute to the forward production of charged pions. If we use the extrapolations to $t \approx 0$ given in Figure 8, we see that the ρ trajectory should dominate at high energies. Since $j(t \lesssim 0) \approx j(\mu^2)$ the pion trajectory is expected to act much like an ordinary pion pole. For small negative t , the pion pole contribution is large since the pole is very close to the physical region. This, in addition to the fact that the πNN coupling is much larger than the ρNN coupling, would indicate that the pion trajectory remains important up to very high energies, at least for small momentum transfers. Since the π trajectory gives rise only to helicity flip, the ρ contribution could in principle be isolated by a study of the no helicity flip part of the amplitude. The quantity $\frac{d\sigma^+}{dt} - \frac{d\sigma^-}{dt}$ gives a measure of the interference between the pion trajectory and the helicity flip parts of the ρ trajectory. Unfortunately, high energy polarized γ -ray beams are necessary to measure this interference.

In the intermediate energy region where both the ρ and π are important, averaging over initial and summing over final polarizations, we have

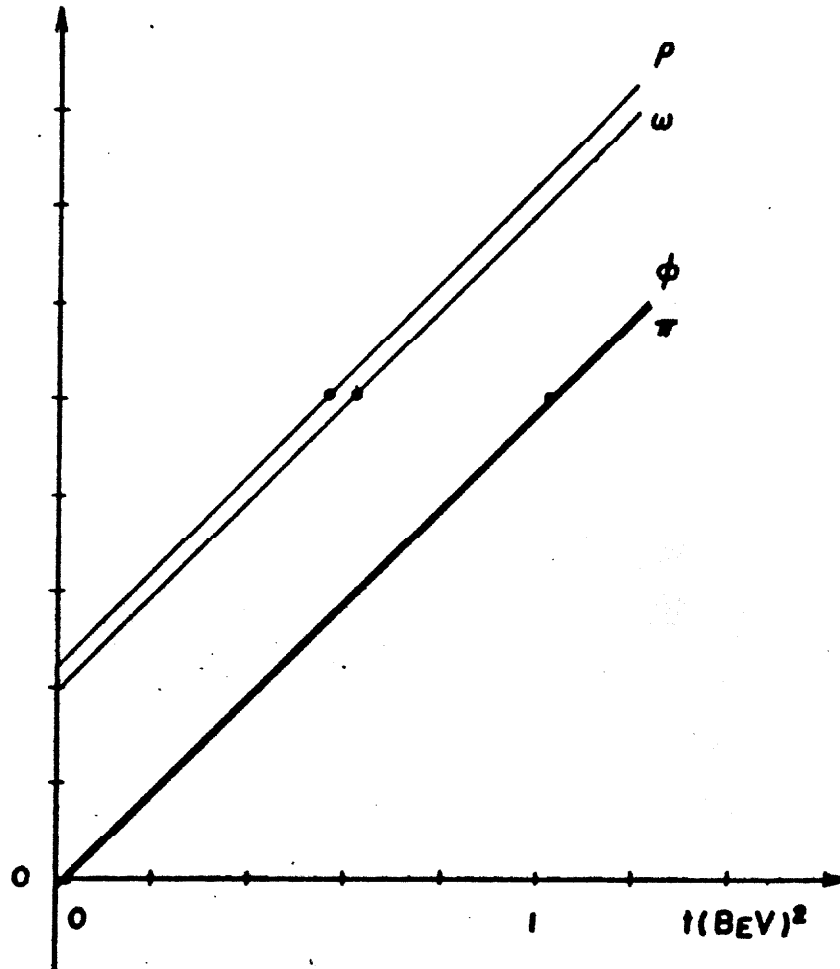


Figure 8: The meson trajectories contributing to photoproduction. The dots at integral spin j represent the mesons. The intersection of the trajectory with the $t = 0$ axis determines the behavior of high energy photoproduction in the forward direction. Although the indicated straight line approximations to the trajectories are not expected to be accurate, the intersections with the $t = 0$ axis may be experimentally measured in other processes like πN and $K N$ scattering. Then, the energy dependence of small angle photoproduction may be predicted and compared with experiment. The present indications are that the differential photoproduction cross-section $\frac{d\sigma}{dt}$ should fall with increasing energy, as compared with the results of an "elementary particle" pole approximation which predicts a constant $\frac{d\sigma}{dt}$ at high energies.

Figure 9

The nucleon trajectories that are used to determine the behavior of large angle photoproduction. The dots at $1/2$ integral spin represent the N and πN resonances. Only the positions of the N and $N_{3/2^+}^*$ have been experimentally confirmed. Some evidence exists indicating that the $N_{1/2^+}^*$ and $N_{1/2^-}^*$ are to be plotted as indicated. The rest of the graph is total conjecture. However, since the same set of trajectories are to be used in π - N scattering, backward π - N scattering and backward photoproduction are expected to have the same energy dependence in the Regge pole approximation.

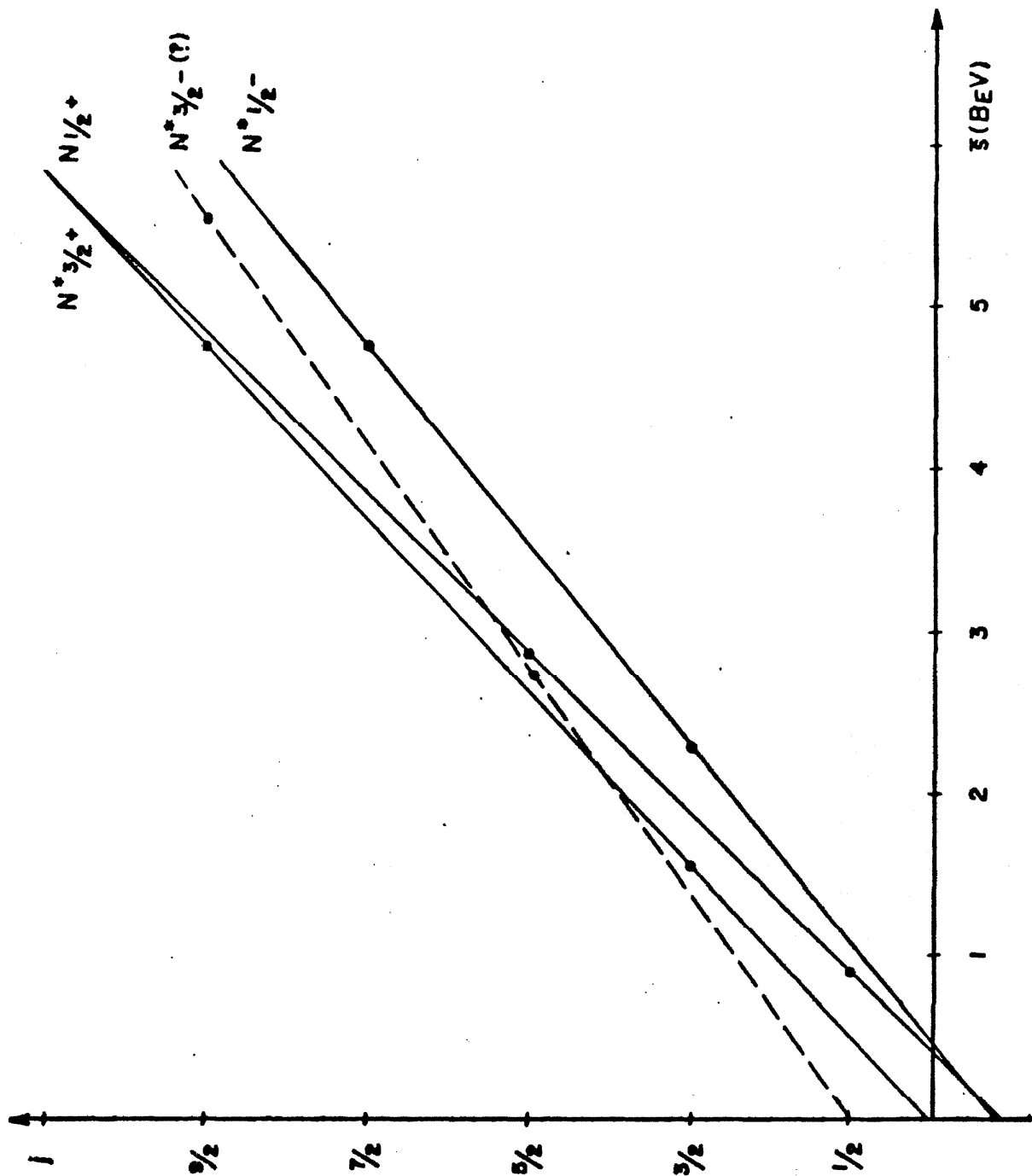


Figure 9

$$4\pi < \frac{d\sigma}{dt} > = \left| \frac{1 - e^{-i\pi j_\rho(t)}}{2 \sin \pi j_\rho(t)} \right|^2 \lambda_\rho^2(t) \left(\frac{s}{s_\rho} \right)^{2j_\rho(t) - 2} +$$

$$\left| j_\pi(t) \frac{1 + e^{-i\pi j_\pi(t)}}{2 \sin \pi j_\pi(t)} \right|^2 \left(\frac{\phi_\pi(t)}{t - \mu^2} \right)^2 \left(\frac{s}{s_\pi} \right)^{2j_\pi(t) - 2}$$

where in terms of our old notation,

$$\lambda_\rho^2(t) = (\eta_\rho^+)^2 + (\phi_\rho^+)^2, \quad \phi_\pi^2(t) = (\phi_\pi^+(t))^2.$$

To test the theory, we take everything essentially constant except for the factors

$$\frac{1}{t - \mu^2}, \quad \left(\frac{s}{s_\rho} \right)^{2j_\rho(t) - 2}, \quad \text{and} \quad \left(\frac{s}{s_\pi} \right)^{2j_\pi(t) - 2}.$$

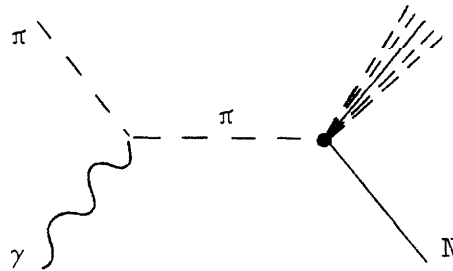
We then plot $\log < \frac{d\sigma}{dt} >$ versus t for fixed s and try to fit the data with the formula given above. In the region where one trajectory dominates, we may write

$$\log < \frac{d\sigma}{dt} > = A + B(2j(t) - 2) \log \left(\frac{s}{s_0} \right)$$

where A and B are constants if we keep away from values of t over which the π pole varies rapidly. We expect $j_\rho(t) \approx 1/2$, $j_\pi(t) \approx 0$ with both j 's decreasing with decreasing t . A plot of $\log < \frac{d\sigma}{dt} >$ versus t for fixed s should give $j(t)$ directly. Because the sign of B is positive, increasing s should make the curve fall. Also of interest would be a graph of $\log < \frac{d\sigma}{dt} >$ versus $\log < \frac{s}{s_0} >$ for fixed t . This should turn out to be a straight line with decreasing slope for decreasing t . For high enough energies, the

ρ dominates giving a cross section for forward scattering that we roughly estimate to be $\frac{d\sigma}{dt} \sim s^{2j_\rho(0)-2} \approx \frac{1}{s}$.

A study of the pion trajectory at high energies is of interest because of its effect on the Drell process. In 1960 Drell indicated that a great number of strongly interacting particles might be produced with a high energy γ ray beam. The important diagram for the process was single pion exchange:



Because the π pole is so close to the physical region, the one pion exchange contribution is expected to be large despite the presence of an electromagnetic coupling. It is through this mechanism that the Stanford linear electron accelerator will produce beams of strongly interacting particles. In the Regge pole approximation, the pion exchange contribution is damped by an amount depending sensitively on $j_\pi(0)$. Because $j_\pi(0)$ is nearly equal to $j_\pi(\mu^2)$, Regge pole effects may become important only at very high energies. Because $j_\pi(0)$ as well as the parameter s_π are presently not well known, a reliable calculation of the damping is impossible.

For π^0 photoproduction, the ρ , ω , and ϕ trajectories contribute. Presumably the ϕ is least important, but this is still uncertain because extrapolations to $t = 0$ are unreliable. The data should be analyzed

in a manner identical to that of π^+ photoproduction. We expect

$$\frac{d\sigma}{dt} \sim s^{2j_\rho(0) - 2} \approx s^{2j_\omega(0) - 2} \approx \frac{1}{s}.$$

In the backward direction the situation is much more complex. There are many trajectories, and in addition to the usual uncertainties of linear extrapolations to $\bar{s} = 0$, there is also the question of whether to include the $N_{3^-/2}^*$ trajectory. There is, however, one interesting conclusion that we may draw. Since the same set of trajectories are to be used in $\pi - N$ scattering, backward $\pi - N$ scattering and backward photoproduction are expected to have the same energy dependence in the Regge pole approximation. If we forget about the $N_{3^-/2}^*$ trajectory, whose existence is presently highly speculative, and assume that the $N_{3^+/2}^*$ trajectory dominates with $j_{N_{3^+/2}^*}(0) \approx 0$, then $\frac{d\sigma}{d\bar{s}} \sim \frac{1}{\bar{s}^2}$.

We might test the hypothesis that $\eta_\rho^+(t)$ and $\phi_\rho^+(t)$ may be factored into

$$\eta_\rho^+(t) = \eta_N^0(t) \gamma_\rho^+(t), \quad \phi_\rho^+(t) = \phi_N^0(t) \gamma_\rho^+(t)$$

by the simultaneous study of

$$\gamma + N \rightarrow \pi + N$$

$$\pi + N \rightarrow \pi + N$$

$$\gamma + N \rightarrow \pi + N_{3^+/2}^*$$

$$\pi + N \rightarrow \pi + N_{3^+/2}^*.$$

The procedure is to experimentally extract the ρ contribution from each process; by assuming that all residues factor, we may obtain a consistency check (see Preface, pg. v). It is interesting to note that only the ρ and π trajectories contribute to the reactions producing $N_{3/2}^*$. Furthermore, at high energies and not too small momentum transfers, the ρ dominates in all processes except $\pi + N \rightarrow \pi + N$.

Finally, we will briefly indicate the effect of a cut in the J plane. Its contribution to the T matrix will have the form of a superposition of poles:

$$T_c = \int_{j_1(t)}^{j_2(t)} g(j,t) \left(\frac{s}{s_c}\right)^j dj$$

where $g(j,t)$ is expected to be a slowly varying function of j compared to $\left(\frac{s}{s_c}\right)^j$. s_c is some convenient constant. To be specific, we take $j_2(t)$, $j_1(t)$ real with $j_2(t) > j_1(t)$. For large s ,

$$T_c \approx g(j_2(t),t) \int_{j_1(t)}^{j_2(t)} \left(\frac{s}{s_c}\right)^j dj \approx g(j_2(t),t) \frac{\left(\frac{s}{s_c}\right)^{j_2(t)}}{\log\left(\frac{s}{s_c}\right)}$$

if $\log\left(\frac{s}{s_c}\right) \gg 1$. If a consideration of cuts is important in the photoproduction process, it is clear that any comparison between theory and experiment becomes extremely difficult.

Appendix

I. The T. Matrix and the Invariant Amplitudes*

The T matrix describing the three processes represented in Figure 1 may be written in the form

$$T_{\beta} = \sum_{i=1}^4 \left[A_i^{(+)}(s, t, \bar{s}) g_{\beta}^{(+)} + A_i^{(-)}(s, t, \bar{s}) g_{\beta}^{(-)} + A_i^{(o)}(s, t, \bar{s}) g_{\beta}^{(o)} \right] M_i$$

where $M_1 = i\gamma_5 \not{\epsilon} \not{K}$

$$M_2 = 2i\gamma_5 (P \cdot \epsilon Q \cdot K - P \cdot K Q \cdot \epsilon)$$

$$M_3 = \gamma_5 (\not{\epsilon} Q \cdot K - \not{K} Q \cdot \epsilon)$$

$$M_4 = 2\gamma_5 (\not{\epsilon} P \cdot K - \not{K} P \cdot \epsilon - iM \not{\epsilon} \not{K})$$

with ϵ denoting the photon polarization and $P \equiv \frac{1}{2} (P_1 + P_2)$. Note that each M_i is gauge invariant. The M 's contain the spin structure of the problem while the isotopic spin information is contained in the g 's, where

$$g_{\beta}^{(+)} = \frac{1}{2} \{ \tau_{\beta}, \tau_3 \} = \delta_{\beta 3}$$

$$g_{\beta}^{(-)} = \frac{1}{2} [\tau_{\beta}, \tau_3]$$

$$g_{\beta}^{(o)} = \tau_{\beta}$$

This isotopic spin form is in part determined by requiring that $\vec{T} \cdot \vec{\pi}$ be invariant under rotations about the 3 axis in isotopic spin space. The specific structure of the g 's then follows by asking that the g 's

* The notation and most of the non-Regge pole results come from
CGLN and Ball, loc. cit.

be either Hermitian or anti-Hermitian. This latter requirement is only a matter of convenience but will result in simple crossing relations for the A's. The index β labels the outgoing pion, while the τ matrices operate on the isotopic spinors of the nucleons. We have 3 g's because this is number of linearly independent matrices necessary to express an arbitrary large product of τ_3 , τ_β , and $\vec{\tau} \cdot \vec{\tau}$ matrices. Table 1 indicates what values are to replace the g's in any given physical process.

Table 1

	$\gamma+p \rightarrow \pi^0+p$	$\gamma+n \rightarrow \pi^0+n$	$\gamma+p \rightarrow \pi^++n$	$\gamma+n \rightarrow \pi^-+p$
g^+	1	1	0	0
g^-	0	0	$\sqrt{2}$	$-\sqrt{2}$
g^0	1	-1	$\sqrt{2}$	$\sqrt{2}$

The unknown dynamics of the problem are contained in the twelve scalar functions $A_i^{(+),(-),2,0}(s,t,\bar{s})$. Crossing symmetry tells us that

$$\begin{aligned}
 & \begin{array}{cc} (+), (0) & (+), (0) \\ A_{1,2,4}(s,t,\bar{s}) = A_{1,2,4}(\bar{s},t,s) & \end{array} \quad \begin{array}{cc} (-) & (-) \\ A_{1,2,4}(s,t,\bar{s}) = -A_{1,2,4}(\bar{s},t,s) & \end{array} \\
 & \begin{array}{cc} (+), (0) & (+), (0) \\ A_3(s,t,\bar{s}) = -A_3(\bar{s},t,s) & \end{array} \quad \begin{array}{cc} (-) & (-) \\ A_3(s,t,\bar{s}) = A_3(\bar{s},t,s) & \end{array}
 \end{aligned} \tag{11}$$

With a spinor normalization of $\bar{u}u = 1$,

$$\frac{d\sigma}{d\Omega}_{c.m.} = \frac{M^2}{16\pi^2} \frac{q}{k} \frac{|T|^2}{s}, \quad \frac{d\sigma}{d\Omega}_{lab} = \frac{1}{(4\pi)^2} \frac{M}{k} \frac{q^2 |T|^2}{(M+k)q - \omega k \cos\theta}$$

II. The Helicity Amplitudes.

The connection between A_i and the helicity amplitudes η^\pm, ϕ^\pm is rather involved. If we define F_i and \mathcal{F}_i by

$$\begin{aligned}
 F_1 &= A_1 + (W - M)A_4 - \frac{t - 1}{2(W - M)} (A_3 - A_4) \\
 &= \frac{8\pi W}{W - M} \frac{\mathcal{F}_1}{(E_2 + M)^{1/2} (E_1 + M)^{1/2}} \\
 F_2 &= -A_1 + (W + M)A_4 - \frac{t - 1}{2(W + M)} (A_3 - A_4) \\
 &= \frac{8\pi W}{W - M} \frac{(E_2 + M)^{1/2}}{(E_1 + M)^{1/2}} \frac{\mathcal{F}_2}{q} \\
 F_3 &= (W - M)A_2 + A_3 - A_4 \\
 &= \frac{8\pi W}{W - M} \frac{\mathcal{F}_3}{(E_2 + M)^{1/2} (E_1 + M)^{1/2} q} \\
 F_4 &= -(W + M)A_2 + A_3 + A_4 \\
 &= \frac{8\pi W}{W - M} \frac{(E_2 + M)^{1/2}}{(E_1 + M)^{1/2}} \frac{\mathcal{F}_4}{q^2}
 \end{aligned} \tag{12}$$

where $W = \sqrt{s}$.

We then have

$$\begin{aligned}
 \eta^- &= \frac{i}{\sqrt{2}} \sin \theta \cos \frac{\theta}{2} (\mathcal{F}_3 + \mathcal{F}_4) \\
 \eta^+ &= -i \sqrt{2} \sin \frac{\theta}{2} (\mathcal{F}_1 + \mathcal{F}_2) - \eta^-
 \end{aligned} \tag{13}$$

$$\phi^+ = \frac{i}{\sqrt{2}} \sin \theta \sin \frac{\theta}{2} (\mathcal{T}_3 - \mathcal{T}_4)$$

$$\phi^- = i \sqrt{2} \cos \frac{\theta}{2} (\mathcal{T}_1 - \mathcal{T}_2) - \phi^+$$

For fixed t large s , we may simplify to obtain

$$\begin{aligned} \eta^- &\rightarrow \frac{i\sqrt{2}}{16\pi} \sqrt{-t} \left[\frac{\mu^2 M}{\sqrt{s}} A_2 + \sqrt{s} (A_3 - A_4) \right] \\ \eta^+ &\rightarrow \frac{-i\sqrt{2}}{8\pi} \sqrt{-t} \left[\frac{M}{\sqrt{s}} A_1 - \frac{(t-1)}{2\sqrt{s}} A_3 + \sqrt{s} A_4 \right] - \eta^- \\ \phi^+ &\rightarrow \frac{-i\sqrt{2}}{16\pi} t \left[\sqrt{s} A_2 + \frac{M}{\sqrt{s}} (A_3 - A_4) \right] \\ \phi^- &\rightarrow \frac{i\sqrt{2}}{8\pi} \left[\sqrt{s} A_1 - \frac{M(t-1)}{\sqrt{s}} A_3 + \frac{Mt}{\sqrt{s}} A_4 \right] - \phi^+ \end{aligned} \quad (14)$$

For fixed \bar{s} large s , the corresponding relationships have already been given in equation 9.

In terms of the helicity amplitudes,

$$\frac{d\sigma^+}{d\Omega_{c.m.}} = \frac{q}{k} \left[|\eta^+|^2 + |\phi^+|^2 \right] \rightarrow |\eta^+|^2 + |\phi^+|^2$$

III. The t Channel: $\gamma + \pi \rightarrow N + \bar{N}$:

The invariant A amplitudes simultaneously describe the three physical processes $\gamma + N \rightarrow \pi + N$, $\gamma + \pi \rightarrow N + \bar{N}$, and $\gamma + \bar{N} \rightarrow \pi + \bar{N}$ (Figure 1). Resonances in any one of these channels effect all three. We will now consider ω , ϕ , ρ and π exchange in the t channel and their influence on forward high energy photoproduction.

It is convenient to introduce P_1' and Q' , the four momenta of the antinucleon and pion. Then, we have $P_1' = -P_1$, $Q' = -Q$. In terms of P_1' and Q' we may write in the c.m. system

$$\begin{aligned} s &\equiv -(K - P_1')^2 = M^2 - 2Ek' - 2pk' \cos \theta' \\ t &\equiv -(Q' + K)^2 = (2E)^2 \\ \bar{s} &\equiv -(P_2 - K)^2 = M^2 - 2Ek' + 2pk' \cos \theta' \end{aligned} \quad (15)$$

where p and k' are the magnitudes of the nucleon and photon momenta, E the total energy of the nucleon, and $\cos \theta' = \frac{\vec{p}_2 \cdot \vec{k}}{pk'}$. In terms of t ,

$$k' = \frac{t - 1}{2\sqrt{t}} \quad p = \frac{1}{2} \sqrt{t - 4M^2}$$

If we write the differential cross section for $\gamma + \pi \rightarrow N + \bar{N}$ in the c.m. system as

$$\frac{d\sigma}{d\Omega} = (p/k') |x_N G y_{\bar{N}}|^2 \quad \text{where}$$

$$\begin{aligned} G &= (\vec{p}_2 \cdot \vec{\epsilon}/p)G_1 + (i\vec{\sigma} \cdot \vec{p}_2 \times \vec{\epsilon}/p)G_2 + (i\vec{\sigma} \cdot \vec{p}_2 \vec{p}_2 \cdot \vec{k} \times \vec{\epsilon}/p^2 k')G_3 + \\ &\quad (i\vec{\sigma} \cdot \vec{k} \times \vec{\epsilon}/k')G_4 \end{aligned}$$

and $x_N, y_{\bar{N}}$ are nucleon and antinucleon Pauli spinors, then the G_i may be decomposed into

$$\begin{aligned}
 G_1 &= -\Sigma_J (J + 1/2) \beta_J^- P_J' (x') \\
 G_2 &= -\frac{1}{2} \Sigma_J \left\{ \alpha_J^- \left[J P_{J+1}'' (x') + (J + 1) P_{J-1}'' (x') \right] \right. \\
 &\quad \left. + (2J + 1) \alpha_J^+ P_J'' (x') \right\} \tag{16}
 \end{aligned}$$

$$\begin{aligned}
 G_3 &= \frac{1}{2} \Sigma_J \left\{ \alpha_J^+ \left[J P_{J+1}'' (x') + (J + 1) P_{J-1}'' (x') \right] \right. \\
 &\quad \left. - (2J + 1) \alpha_J^- P_J'' (x') - (2J + 1) \beta_J^+ P_J' (x') \right\} \\
 G_4 &= -\frac{1}{2} \Sigma_J \left\{ \alpha_J^+ \left[J P_{J+1}'' (x') + (J + 1) P_{J-1}'' (x') \right] - (2J + 1) \alpha_J^- P_J'' (x') \right\}
 \end{aligned}$$

Isotopic spin indices have been suppressed. The amplitudes α and β , which are functions of t , have the following physical significance: α_J^+ and β_J^+ lead to triplet nucleon-antinucleon final states of parity $(-1)^J$ and total angular momentum J . Triplet final states of parity $(-1)^{J+1}$ are represented by α_J^- , while β_J^- leads to a singlet final state of angular momentum J and parity $(-1)^{J+1}$.

For the $\bar{N}N$ system,

$$\text{parity } P = (-1)^{L+1}$$

$$\text{G-parity } G = (-1)^{L+S+T}$$

where L, S, T are the total orbital angular momentum, spin, and isotopic spin of the $\bar{N}N$. Hence, both the ρ and ω contribute only to the triplet

state, while the π must lead to a singlet system. Consequently, the ρ and ω excite α_1^+ and β_1^+ , while the π contributes through β_0^- . In general, a particle of spin J on the ρ or ω trajectory would enhance α_J^+ and β_J^+ , while a particle on the π trajectory would contribute only through β_J^- .

In terms of A_i ,

$$\begin{aligned} G_1 &= (k' p / 16\pi E) [A_1 + t A_2] \\ G_2 &= - (k' p / 4\pi) A_3 \\ G_3 &= (M - E) k' / 8\pi E [A_1 + \sqrt{t} A_4] \\ G_4 &= (k' / 16\pi E) [2MA_1 - t A_4] \end{aligned} \tag{17}$$

High energy photoproduction will be governed by A_1 for large s or equivalently, G_1 for large x' (see equation 15). As the series given in equation 16 do not converge for large values of x' , a Sommerfeld-Watson transformation will be performed. The summation over J will be replaced by a contour integral in the complex angular momentum plane. The contour will then be deformed to obtain the required asymptotic expansion for the scattering amplitude. In deforming the contour, contributions to the G's from poles and cuts in the J plane will be picked up. As a crude approximation we will consider the pole terms corresponding to known physical particles. However, it is most probably that a satisfactory explanation of high energy photoproduction cannot be obtained in this fashion and that cuts in the J plane cannot

be disregarded. Since there is presently no information about the positions of, or the discontinuities across, the cuts, we will confine ourselves to the Regge pole approximation.

A. The π Trajectory:

With the help of equations 16 and 17,

$$A_2^{(+)(-)(o)} = \frac{32\pi G_1^{(+)(-)(o)}}{(t - \mu^2) \sqrt{t - 4M^2}} \rightarrow$$

$$\frac{1}{t - \mu^2} j_\pi(t) \frac{1 + e^{-i\pi j_\pi(t)}}{2 \sin \pi j_\pi(t)} a(t)^{(+)(-)(o)} \left(\frac{s}{s_\pi}\right)^{j_\pi(t)-1}$$

where we have taken the liberty of absorbing the $\frac{1}{\sqrt{t - 4M^2}}$ kinematic

factor into $a(t)$ since we know on physical grounds that A_2 cannot blow up at $t = 4M^2$. There is still some question as to what should be done with the $\frac{1}{t - \mu^2}$. As a guide we examine A_2 in the pole approximation (Figure 6). The result is

$$A_2^{(+)(o)} = \frac{-eg}{t - \mu^2} \left(\frac{1}{s - M^2} + \frac{1}{\bar{s} - M^2} \right) = \frac{eg}{(s - M^2)(M^2 + \mu^2 - s - t)}$$

$$A_2^{(-)} = \frac{-eg}{t - \mu^2} \left(\frac{1}{s - M^2} - \frac{1}{\bar{s} - M^2} \right) = \frac{eg}{t - \mu^2} \frac{2s + t - 2M^2 - \mu^2}{(s - M^2)(M^2 + \mu^2 - s - t)}$$

where e and g are the rationalized and renormalized electric charge and pion-nucleon coupling constant: $\frac{e^2}{4\pi} = \frac{1}{137}$, $\frac{g^2}{4\pi} \approx 14$. Note that $\frac{1}{t - \mu^2}$, the kinematic pole (the μ^2 refers to the mass of the external π),

is only present in $A_2^{(-)}$, and that for large s , $A^{(+)(o)} \ll A_2^{(-)}$ so

that to the approximation in which we are working, $a^{(+)(o)}(t) \equiv 0$.

In $A_2^{(-)}$, for $t \approx \mu^2$ the zero of $j_\pi(t)$ in the numerator just cancels the pole $\frac{1}{\sin \pi j_\pi(t)}$, leaving the kinematic $\frac{1}{t - \mu^2}$ factor that is responsible for the observed retardation effect in photoproduction.

Since the amplitudes A_i receive contributions, in the sense of dispersion relations, only from real intermediate states, the "pion pole" must appear in our formalism as a kinematic singularity (the coupling of 2 real π 's to a γ must vanish). If we relax the condition of gauge invariance, then real π intermediate states will become possible, but new amplitudes B_i will have to be introduced to describe the process. The B_i will contain no kinematic singularities and will receive ordinary pole contributions from real intermediate π 's. If we now impose gauge invariance on the B_i , not all the B_i will be independent and equations of the form $(s - \bar{s}) B_2 = 2(t - \mu^2) B_3, \dots$ might result. In then relating the A_i to the B_i , we would find the appearance of a $\frac{1}{t - \mu^2}$ pole that is not related to any real intermediate state. The $\frac{1}{t - \mu^2}$ term may thus be looked upon as a kinematic effect or a result of gauge invariance.*

* For details concerning gauge invariance and the Mandelstam representation, see Ball, loc. cit.

Also at $t \approx \mu^2$, the unknown function $a^{(-)}(t)$ is determined to be

$$a^{(-)}(\mu^2) = \frac{-2\pi e g}{s\pi}$$

Using equation 14 we obtain the helicity amplitudes in terms of $A_2^{(-)}$:

$$\eta^- \sim \frac{A_2^{(-)}}{\sqrt{s}} \sim s^{j_\pi(t)-3/2}, \quad \eta^+ = -\eta^-$$

$$\phi^+ \sim \sqrt{s} A_2^{(-)} \sim s^{j_\pi(t)-1/2}, \quad \phi^- = -\phi^+$$

$$\text{and } \frac{d\sigma^+}{d\Omega} \rightarrow |\phi^+|^2 \sim s^{2j_\pi(t)-1}$$

which then directly verifies the claims made in our early discussion of the pion trajectory (II.C.).

B. The ρ Trajectory:

Here we find:

$$A_1^{(0)} = -\zeta_\rho(t) t b(t) \left(\frac{s}{s_\rho}\right)^{j_\rho(t)-1} \quad A_3^{(0)} = 0$$

$$A_2^{(0)} = \zeta_\rho(t) b(t) \left(\frac{s}{s_\rho}\right)^{j_\rho(t)-1} \quad A_4^{(0)} = \zeta_\rho(t) c(t) \left(\frac{s}{s_\rho}\right)^{j_\rho(t)-1}$$

This may also be compared with the pole approximation:*

$$\begin{aligned} A_1^{(0)} &= \frac{t^\gamma \rho \pi \gamma^\gamma 2\rho_{NN}}{t - m_\rho^2} & A_3^{(0)} &= 0 \\ A_2^{(0)} &= -\frac{\gamma \rho \pi \gamma^\gamma 2\rho_{NN}}{t - m_\rho^2} & A_4^{(0)} &= -\frac{\gamma \rho \pi \gamma^\gamma 1\rho_{NN}}{t - m_\rho^2} \end{aligned} \quad (18)$$

* M. Gordin, D. Iurie', and A. Martin; Nuovo Cimento 18, 933 (1960)

where the exchange diagram has been written as

$$(\rho\pi\gamma \text{ vertex}) \times \text{propagator} \times (\rho\pi\pi \text{ vertex}) = \gamma_{\rho\pi\gamma} \epsilon_{\alpha}^{(\gamma)} K_{\beta}^Q \epsilon_{\alpha\beta\gamma\delta} \frac{-1}{t - m_{\rho}^2}$$

$$\bar{u}(\vec{p}_2) \gamma_{1\rho\pi\pi} \gamma_{\delta} + \gamma_{2\rho\pi\pi} \frac{i[\not{p}_2 - \not{p}_1, \gamma_{\delta}]}{2} \tau_{\mu} u(p_1) \quad *$$

The index μ on the τ_{μ} labels the outgoing pion.

In the vicinity of $t = m_{\rho}^2$, breaking j_{ρ} up into its real and imaginary parts

$$j_{\rho}(t) \approx 1 + (t - m_{\rho}^2) \epsilon_{\rho} + i I_{\rho}$$

we get

$$\frac{b(m_{\rho}^2)}{\pi \epsilon_{\rho}} = \gamma_{\rho\pi\gamma} \gamma_{2\rho\pi\pi}$$

$$\frac{c(m_{\rho}^2)}{\pi \epsilon_{\rho}} = \gamma_{\rho\pi\gamma} \gamma_{1\rho\pi\pi}$$

$$I_{\rho}/\epsilon_{\rho} = m_{\rho} \Gamma_{\rho}$$

where near the pole we have used $\frac{1}{t - m_{\rho}^2 + i m_{\rho} \Gamma_{\rho}}$ instead of

$$\frac{1}{t - m_{\rho}^2} \text{ in equation 18.}$$

* The τ_{μ} comes from assuming an isotopic spin interaction of the form $A(\vec{\rho} \cdot \vec{\pi})(\vec{\rho} \cdot \vec{N} \vec{\tau} \cdot \vec{N})$. Here we may say that the electromagnetic field behaves as a scalar in isotopic spin space and that the total interaction is invariant under rotations in this space. The isoscalar photons which this field A creates will then have definite properties under G parity identical to those of the ω and will contribute only to $A_i^{(0)}$. Another interaction $(\vec{\rho} \times \vec{\pi})_z A \equiv (\vec{\rho} \times \vec{\pi}) \cdot \vec{A}$ which treats A as an isovector and would contribute to $A_i^{(-)}$, gives zero since under G parity an isovector γ behaves as a ρ , and the $\rho\rho\pi$ vertex vanishes.

Using equations 14 we immediately arrive at the ρ contribution to $\frac{d\sigma}{dt}$ given in equation 4.

C. The ω and ϕ Trajectories

The ω and ϕ are handled exactly as the ρ . Due to isotopic spin differences, however, they contribute to $A_i^{(+)}$ instead of $A_i^{(0)}$.

As a side comment not dealing with very large photon energies we would like to note that besides the ρ , ω , and ϕ , non-resonating (nr) intermediate states may be easily included. For example, in the ρ channel instead of equation 18 it would be more accurate to write

$$A_1^{(0)} = t\gamma_{\rho\pi\gamma} \left\{ \frac{\gamma_{2\rho NN}}{t - m_\rho^2} - \frac{\gamma_{(nr)\pi\gamma}}{\gamma_{\rho\pi\gamma}} \gamma_{2(nr)NN} \right\}$$

$$A_2^{(0)} = -\gamma_{\rho\pi\gamma} \left\{ \frac{\gamma_{2\rho NN}}{t - m_\rho^2} - \frac{\gamma_{(nr)\pi\gamma}}{\gamma_{\rho\pi\gamma}} \gamma_{2(nr)NN} \right\}$$

where everything inside the braces is known from measurements of the nucleon form factors except $\frac{\gamma_{(nr)\pi\gamma}}{\gamma_{\rho\pi\gamma}}$. $\gamma_{(nr)\pi\gamma}$ represents some kind

of an unknown effective coupling between non-resonating intermediate states and the $\pi\gamma$. Attempts to fit certain photoproduction data

using the isovector nucleon form factors have been carried out by

Hohler and Dietz.⁽⁵⁾ Unfortunately, the expressions they use for

$A_i^{(0)}$ are incorrect.* If these non-resonant effects are observable,

* In terms of our notation, their solutions for $A_{1,2}^{(0)}$ are

$A_1^{(0)} = \gamma_{\rho\pi\gamma} \left\{ t \frac{\gamma_{2\rho NN}}{t - m_\rho^2} - \gamma_{2(nr)NN} \right\}$, $A_2^{(0)} = \frac{-\gamma_{\rho\pi\gamma} \gamma_{2\rho NN}}{t - m_\rho^2}$. Physically this makes no sense because the coupling $\gamma_{(nr)\pi\gamma}$ is missing. Formally it must be in error because it does not satisfy $G_1 \propto A_1 + tA_2 = 0$. Their mistake comes in using an approximate unitarity relation in a region where it no longer applies.

they should show up at large t (for example, backward scattering with incident γ energies of 1 to 1.5 BeV).

IV. The \bar{s} Channel: $\gamma + \bar{N} \rightarrow \pi + \bar{N}$.

We wish to find the effects of particles (or resonances) exchanged in the \bar{s} channel on high energy scattering in the s channel. First, we will put the particles in the s channel. Then, by making use of the crossing relations given in equations 11 and switching $s \leftrightarrow \bar{s}$ we will end up with amplitudes representing particle exchange in the \bar{s} channel.

A. The N Trajectory:

The functions \mathcal{F}_i defined in equations 12 admit the following angular momentum decomposition.

$$\begin{aligned}\mathcal{F}_1 &= \sum_{\ell=0} [\ell M_{\ell+} + E_{\ell+}] P'_{\ell+1}(x) + [(\ell+1)M_{\ell-} + E_{\ell-}] P'_{\ell-1}(x) \\ \mathcal{F}_2 &= \sum_{\ell=1} [(\ell+1)M_{\ell+} + \ell M_{\ell-}] P'_\ell(x) \\ \mathcal{F}_3 &= \sum_{\ell=1} [E_{\ell+} - M_{\ell+}] P''_{\ell+1}(x) + [E_{\ell-} + M_{\ell-}] P''_{\ell-1}(x) \\ \mathcal{F}_4 &= \sum_{\ell=1} [M_{\ell+} - E_{\ell+} - M_{\ell-} - E_{\ell-}] P''_\ell(x)\end{aligned}\tag{19}$$

$M_{\ell\pm}$ and $E_{\ell\pm}$ are energy dependent amplitudes describing transitions initiated by magnetic and electric multipole radiation leading to final states of orbital angular momentum ℓ and total angular momentum $\ell \pm 1/2$.

Consulting Table 2 we see that the nucleon contributes to M_{1-} and that other particles on its trajectory would excite E_{3-} , M_{3-} ; E_{5-} , M_{5-} ; ... The amplitudes $E_{\ell+}$ and $M_{\ell+}$ may not be neglected,

Table 2

<u>Multipole</u>	<u>Total J</u>	<u>Total parity</u>	<u>ℓ of π-N system</u>	<u>Amplitude effected</u>
E1	1/2	-	0	E_{0+}
"	3/2	-	2	E_{2-}
M1	1/2	+	1	M_{1-}
"	3/2	+	1	M_{1+}
E2	3/2	+	1	E_{1+}
"	5/2	+	3	E_{3-}
M2	3/2	-	2	M_{2-}
"	5/2	-	2	M_{2+}
E3	5/2	-	2	E_{2+}
"	7/2	-	4	E_{4-}
M3	5/2	+	3	M_{3-}
"	7/2	+	3	M_{3+}
E4	7/2	+	3	E_{5+}
"	9/2	+	5	E_{5-}
M4	7/2	-	4	M_{4-}
"	9/2	-	4	M_{4+}
E5	9/2	-	4	E_{4+}
"	11/2	-	6	E_{6-}
M5	9/2	+	5	M_{5-}
"	11/2	+	5	M_{5+}

however, because of the symmetry

$$\eta_{-}^{+}(W) = \eta_{-}^{+}(-W), \quad \phi_{-}^{+}(W) = -\phi_{-}^{+}(-W) \quad (8)$$

which, in terms of the multipole states, takes the rather complex form:

$$\begin{aligned} M_{\ell+}(W) &= \frac{1}{\ell+1} \left[(\ell+2) M_{(\ell+1)-}(-W) + E_{(\ell+1)-}(-W) \right] \\ E_{\ell+}(W) &= \frac{1}{\ell+1} \left[M_{(\ell+1)-}(-W) - \ell E_{(\ell+1)-}(-W) \right] \end{aligned} \quad (20)$$

If we use equations 20 to eliminate $M_{\ell+}$ and $E_{\ell+}$ from equations 19 we obtain

$$\begin{aligned} F_1(W) &= F_2(-W) \\ F_2(W) &= \sum \ell M_{\ell-}(W) P_{\ell}'(x) + \sum \left[(\ell+1) M_{\ell-}(-W) + E_{\ell-}(-W) \right] P_{\ell-1}'(x) \\ F_3(W) &= F_4(-W) \\ F_4(W) &= - \sum \left[M_{\ell-}(W) + E_{\ell-}(W) \right] P_{\ell}''(x) + \sum \left[M_{\ell-}(-W) + E_{\ell-}(-W) \right] P_{\ell-1}''(x) \end{aligned} \quad (21)$$

The relations

$$F_1(W) = F_2(-W), \quad F_3(W) = F_4(-W) \quad *$$

may also be directly verified with the help of equations 12 along with

$$F_1(W) = -F_2(-W)$$

$$F_3(W) = F_4(W)$$

When we eventually switch s and \bar{s} , x will become \bar{x} , the scattering angle in the \bar{s} channel, and $P_{\ell}(x)$ above will be replaced by $P_{\ell}(\bar{x})$.

* These equations also follow directly from the symmetry

$$\eta_{-}^{+}(W) = \eta_{-}^{+}(-W), \quad \phi_{-}^{+}(W) = -\phi_{-}^{+}(-W)$$

and equations 13.

High energy backward scattering in the s channel will mean large \bar{x} so that $P_\ell(\bar{x}) \sim \bar{x}^\ell \sim s^\ell$. Consequently, it will be legitimate at this point to forget about terms multiplying $P_{\ell-1}'$ in \mathcal{F}_1 and \mathcal{F}_2 , and those multiplying $P_{\ell-1}''$ in \mathcal{F}_3 and \mathcal{F}_4 .

Using equations 12 we may solve for F_i in terms of \mathcal{F}_i , and A_i in terms of F_i . We obtain

$$\begin{aligned} A_1 &= \frac{1}{2W} \left\{ W_+ F_1 + W_- F_2 - M(t - \mu^2) \left[\frac{F_3}{W_-} - \frac{F_4}{W_+} \right] \right\} \\ A_2 &= \frac{1}{2W} \{ F_3 - F_4 \} \\ A_3 &= \frac{1}{2W} \left\{ F_1 + F_2 - \frac{(s - \bar{s})}{2} \left[\frac{F_3}{W_-} - \frac{F_4}{W_+} \right] \right\} \\ A_4 &= \frac{1}{2W} \left\{ F_1 + F_2 - \frac{(t - \mu^2)}{2} \left[\frac{F_3}{W_-} - \frac{F_4}{W_+} \right] \right\} \\ A_3 + A_4 &= \frac{1}{2W} (s - M^2) \left[\frac{F_3}{W_-} - \frac{F_4}{W_+} \right] \end{aligned} \tag{22}$$

where $W_\pm = W \pm M$. If we now apply the crossing relations given in equations 11, go to the limit of large \bar{x} by performing a Sommerfeld-Watson transformation and, pick up only the nucleon trajectory as a singularity in the angular momentum plane, we arrive at equations 10.

In the pole approximation, suppressing isospin indices,

$$\begin{aligned} A_1 &\rightarrow \pm \frac{eg}{2} \frac{1}{\bar{s} - M^2} = \pm \frac{eg}{2} \left\{ \frac{1}{\sqrt{\bar{s}} - M} + \frac{1}{-\sqrt{\bar{s}} - M} \right\} \frac{1}{2M} \\ A_2 &\rightarrow \pm \frac{eg}{s} \frac{1}{\bar{s} - M^2} \\ A_3 &\rightarrow \pm R_3 \frac{1}{\bar{s} - M^2} \end{aligned}$$

$$A_4 \rightarrow \pm R_4 \frac{1}{s - M^2}$$

where the upper (lower) sign is to be taken for A's even (odd) under crossing.

$$R_3^{(+)(-)} = R_4^{(+)(-)} = -\frac{1}{2} g (\mu'_p - \mu_n)$$

$$R_3^{(0)} = R_4^{(0)} = -\frac{1}{2} g (\mu'_p + \mu_n)$$

with $\mu'_p = 1.78 \frac{e}{2M}$ and $\mu_n = -1.91 \frac{e}{2M}$.

Comparing this with equations 10 we find

$$\frac{b_N^{(+)(-)(0)}(M)}{\pi \epsilon_+} = \frac{b_N^{(+)(-)(0)}(-M)}{\pi \epsilon_-} = \frac{eg}{4M^2}$$

$$\frac{a_N^{(+)(-)(0)}(M)}{\pi \epsilon_+} = \frac{a_N^{(+)(-)(0)}(-M)}{\pi \epsilon_-} = \frac{R_3^{(+)(-)(0)}}{2M} - \frac{eg}{8M^2}$$

where

$$\alpha_+ \equiv j(\sqrt{s}) - 1/2 \approx (\sqrt{s} - M)\epsilon_+ \text{ for } \sqrt{s} \approx M$$

$$\alpha_- \equiv j(-\sqrt{s}) - 1/2 \approx (-\sqrt{s} - M)\epsilon_- \text{ for } \sqrt{s} \approx -M$$

~~Similar formulae may be written down at $s \approx 1690$ MeV, the mass of the $N_{1/2}^{**+}$ which is the next resonance on the nucleon trajectory.~~

B. The $N_{3/2}^{*+}$ and $N_{1/2}^{*-}$ Trajectory:

Consulting Table 2, we find that the $N_{1/2}^{*-}$ trajectory contributes through $E_{2-}, M_{2-}; E_{4-}, M_{4-}; \dots$ This means that the results obtained for the nucleon case may be used directly with only a change of signature.

Again from Table 2, the $N_{3/2}^{*+}$ trajectory excites $E_{1+}, M_{1+}; E_{3+}, M_{3+}; \dots$ We now must eliminate the $E_{\ell-}$ and $M_{\ell-}$ amplitudes from

equations 19 using equations 20. The result is

$$\begin{aligned}\mathcal{F}_1(W) &= \sum \left[\mathcal{M}_{\ell+}(W) + \mathcal{E}_{\ell+}(W) \right] P'_{\ell+1} + \dots \\ \mathcal{F}_2(W) &= \mathcal{F}_1(-W) \\ \mathcal{F}_3(W) &= \sum \left[\mathcal{E}_{\ell+}(W) - \mathcal{M}_{\ell+}(W) \right] P''_{\ell+1} + \dots \\ \mathcal{F}_4(W) &= \mathcal{F}_3(-W)\end{aligned}$$

Comparing this with equations 21 we see that whereas in the N case the argument $-W$ appeared in $\mathcal{F}_{1,3}$, now it appears in $\mathcal{F}_{2,4}$. From this the results of III.B. immediately follow.

I. DETERMINATION OF A THEORY FOR LEPTONIC K DECAY

One of the outstanding problems in the theory of weak interactions consists of finding a unifying principle for the strangeness changing and non-strangeness changing decays. Attempts to use a universal Fermi interaction or to generalize the idea of a conserved non-strangeness changing vector current have not been fruitful in the sense that an understanding of the experimental data has not been obtained.⁽¹⁾ Furthermore, the ideas developed in attempting to explain the striking success of the Goldberger-Treiman formula in $\pi \rightarrow \mu \nu$ decay⁽²⁾ have not been carried over successfully into the theory of K decays.⁽³⁾ Many of the present difficulties may well stem from our inability to give operational definitions to such concepts as a "partially-conserved current" and "universal interaction". In an attempt to sharpen our understanding of these terms, we have considered the leptonic decays of the K^+ .

The assumption is made that the $K^+ \rightarrow \ell^+ + \nu + \pi^0$ interaction is of the vector form, in which case we may write for the decay amplitude:

$$\langle \ell^+ \nu \pi^0 | K^+ \rangle = i \frac{G}{\sqrt{2}} \left(\frac{m_\nu}{E_\nu} \frac{m_\ell}{E_\ell} \right)^{\frac{1}{2}} \bar{\nu} \gamma_\alpha (1 + \gamma_5) \ell^+ \langle \pi^0 | s_\alpha^V | K^+ \rangle \quad (1)$$

where s_α^V is the strangeness changing vector current, and G is the weak interaction constant equal to 1.4×10^{-49} ergs \times cm³. By Lorentz invariance arguments, the matrix element $\langle \pi^0 | s_\alpha^V(0) | K^+ \rangle$ may be thrown into the form

$$\frac{1}{\sqrt{2}} \langle \pi^0 | s_\alpha^V(0) | K^+ \rangle = \frac{1}{2} (4 E_K E_\pi)^{-\frac{1}{2}} \left[(p_K + p_\pi)_\alpha f_+(s) + (p_K - p_\pi)_\alpha f_-(s) \right] \quad (2)$$

where $s = -(p_K - p_\pi)^2$. The four-momenta of the K and π are p_K and p_π .

Using causality arguments, one can show that $f_+(s)$ and $(m_K^2 - m_\pi^2) f_+(s) + s f_-(s)$ satisfy subtracted dispersion relations.

It is not difficult to show that f_+ receives contributions (in the sense of dispersion theory) only from P-wave intermediate states. Also, since the matrix element $\langle \pi^0 | \partial_\alpha s_\alpha^V(0) | K^+ \rangle$ of the divergence of s_α^V is proportional to $(m_K^2 - m_\pi^2) f_+(s) + s f_-(s)$, it is precisely this combination of form factors that receives contributions from S-wave intermediate states. We now explicitly take into account the K^* ($K\pi$ spin 1⁻ resonance at 884 MeV), the only known particle or resonance that will contribute to our form factors.* Hence, we write

$$f_+(s) = \gamma \left\{ \frac{1}{1 - \frac{s}{M^2}} + v(s) \right\}, \quad (3)$$

$$\langle \pi^0 | \partial_\alpha s_\alpha^V(0) | K^+ \rangle \propto \Delta m^2 f_+(s) + s f_-(s) = \gamma \Delta m^2 d(s) \quad (4)$$

where $\Delta m^2 = m_K^2 - m_\pi^2$, M is the mass of the K^* , and γ is a coupling constant that measures the strength of the $K^* K\pi$ interaction.

Because we do not know of any zero mass particle that would give rise to poles in our form factors, we find

$$f_+(s) = \gamma \left\{ d(0) + \frac{s}{M^2} \frac{1}{1 - \frac{s}{M^2}} + v(s) - v(0) \right\} \quad (5)$$

$$\text{and } f_-(s) = -\gamma \frac{\Delta m^2}{M^2} \left\{ \frac{1}{1 - \frac{s}{M^2}} + \frac{M^2}{s} (v(s) - v(0)) - \frac{M^2}{s} (d(s) - d(0)) \right\}. \quad (6)$$

We now make the assumption that the current s_α^V is "partially conserved", by which we mean that $d(s)$ is slowly varying and $|d(s)| \ll 1$,

* Later in this paper we discuss the effects of other possible $K\pi$ resonances.

in the physical region for s . This will justify neglecting the term $-\frac{M^2}{s} (d(s) - d(0))$ in the expression for $f_-(s)$. Note that this definition for the partial conservation of s_α^V differs from the ones usually adopted. Previously, the partial conservation of s_α^V has been taken to mean $\partial_\alpha s_\alpha^V = 0$ in the limit of some higher symmetry where baryon mass difference and meson mass difference vanish.⁽⁴⁾ Alternative definitions have stipulated that $\langle \pi^0 | \partial_\alpha s_\alpha^V | K^+ \rangle \rightarrow 0$ as $s \rightarrow \infty$.⁽⁵⁾ Since neither one of these latter two conditions is directly measurable in any decay experiment, we have chosen to redefine the concept of a partially conserved current.

In order to obtain an expression for f_+ and f_- that may be easily compared with experiment, we will make the rather crude approximation that $v(s) - v(0)$ is proportional to s . We may then write

$$f_+(s) = \lambda \left\{ \frac{s}{M^2} \frac{1}{1 - \frac{s}{M^2}} + \delta \right\} \quad (7)$$

and

$$f_-(s) = -\lambda \frac{\Delta m^2}{M^2} \frac{1}{1 - \frac{s}{M^2}}, \quad (8)$$

where $\delta = \frac{d(0)}{1 + M^2 \frac{dv(s)}{ds}} \Big|_{s=0} \ll 1$ and $\lambda = \gamma \left(1 + M^2 \frac{dv(s)}{ds} \right) \Big|_{s=0}$. *

We now have a two parameter theory. λ may be determined from the known K_{e3}^+ decay rate, while δ should follow from the observed $K_{\mu 3}^+/K_{e3}^+$ branching ratio.

* After the completion of this work, the author learned of a paper that obtained essentially this same theory from a slightly different point of view. See N. Brene, L. Egardt, B. Qvist, and D. A. Geffen, Nuclear Physics 30, 399 (1962). However, at the time their paper was written, the data of Brown et. al. and Dobbs et. al. was not available.

II. PREDICTIONS AND EXPERIMENTAL CONFIRMATIONS OF THE THEORY

In Figure 1 we have plotted the branching ratio $K_{\mu 3}/K_{e 3}$ vs. δ . The curve is flat except for a very sharp rise near $\delta = 0$. The structure of this spike is a result of the combined hypotheses of a partially conserved current and "dominating" K^* pole. In the region of δ near the peak, not only the branching ratio, but also the spectra of all the particles, along with the longitudinal polarization of the μ , are extremely sensitive functions of δ . The size of δ should be compared with the pole term which has strength 1. A strictly conserved current would mean $\delta = 0$, a theoretically impossible situation ($\delta = 0$ also gives an incorrect branching ratio). Regardless of the value of δ , we may say in general that $K_{\mu 3}^+/K_{e 3}^+ \leq 0.95$. The measured branching ratio is 0.96 ± 0.15 . This gives $\delta = -0.05^{+0.025}_{-0.05}$. Figure 2 shows some typical f_+ 's. Note that this form factor goes through zero in the physical region. Using the known rate for $K_{e 3}^+$ decay, we may find λ^2 as a function of δ . The result is given in Figure 3. Figure 4 shows the rate for $K_{e 3}$ as a function of δ , λ being held fixed.

If the theory is correct, it should be possible to fit both the π^0 and μ^+ energy spectra in $K_{\mu 3}^+$ decay by picking some value of δ in the range $-0.025 \leq \delta \leq -0.1$. Let us therefore look at Figures 5 and 6 where the data from the experiment of Brown et al.⁽⁶⁾ is displayed.

We see that the constant form factors $(\frac{f_-}{f_+} = \xi = -9)$ used by Dobbs et al. and Boyarski et al.⁽⁷⁾ in their experiments cannot possibly fit either the observed π^0 or μ^+ energy distributions as

measured by Brown et al.. The curve corresponding to $\delta = -0.065$ gives reasonable agreement with experiment. Note that Brown et al. use two parameters in their fit while we use one. We find that δ comes out small compared to one, as our theory predicts.

Using the δ obtained from the experiment of Brown et al., we may compute what we would expect Dobbs et al. and Boyarski et al. to find in their experiments. The result is given in Figure 7. Clearly, the form factors determined by Brown et al. do not fit the data of Dobbs et al. and Boyarski et al., while $\delta = -0.065$ gives a result consistent with experiment.

Because Dobbs et al. and Boyarski et al. measure only the upper part of the μ spectrum, while Brown et al. measure the π^0 energy spectrum and the bottom part of the μ spectrum, it is possible that the data of these three groups are not in contradiction. A contradiction will arise only if we assume that the form factors are essentially constant. Figures 8 and 9 give theoretical curves (without experimental biases) for the μ^+ and π^0 energy spectra.

Using the model with a fixed value for δ , the μ longitudinal polarization spectrum may be computed. Figure 10 gives some typical polarization curves. For large μ kinetic energies ($T_\mu > 110$ MeV), the polarization comes out negative for all reasonable δ (all values of δ compatible with the $K_{\mu 3}/K_{e 3}$ branching ratio). For intermediate values of T_μ ($35 \text{ MeV} < T_\mu < 75 \text{ MeV}$), the polarization is positive for all reasonable δ . For $T_\mu < 75 \text{ MeV}$, the polarization can be either positive or negative. An experiment measuring the polarization of high-energy μ 's has been proposed and carried out. The data are now being analyzed.

We would like to emphasize that certain very sensitive quantities, like the polarization of the μ in $K_{\mu 3}^+$ decay or the π^0 energy spectrum in $K_{e 3}^+$ decay, will not be very well determined within the framework of our approximations. Quadratic terms in s should also be included if we expect good agreement with experiment.

If we introduce a particle X to mediate the weak interactions, then we may summarize its effect by a change of form factors.

$$f_{-} \rightarrow f_{-} - \frac{\Delta m^2}{M_x^2} \frac{f_{+}}{1 - \frac{s}{M_x^2}} \quad , \quad f_{+} \rightarrow \frac{1}{1 - \frac{s}{M_x^2}} f_{+} \quad ,$$

where M_x is the mass of the X . Because we lack detailed knowledge of $v(s)$ and $d(s)$, the leptonic decays of the K meson seem to be a poor place for isolating the effects of the X . Figure 11 gives some indication of the size of X effects.

The concept of a universal Fermi interaction has never been very well defined. For example, to test for universality in $K_{\ell 3}^+$ decay, it has been customary to consider $G f_{+}(0) = S_v$ as an effective coupling constant. Since it turns out that $S_v^2 \ll G^2$, a universal form for the interaction is not apparent. However, if $f_{+}(s)$ is rapidly varying with s , then this test for universality may not be fair. Perhaps we should evaluate $G f_{+}(s)$ at a different value of s when forming S_v and making our comparison with G . The slowness of the leptonic decay of the K^+ might then be explained on the basis of a partially-conserved current. The rate is slow because the matrix element is of the order of $\langle \pi^0 | \partial_{\alpha} s_{\alpha}^V | K^+ \rangle$ which is a small quantity because s_{α}^V is partially conserved.

In concluding this section on K^+ decay, we would like to re-emphasize that a partially conserved current implies a profound deviation from what would be expected on the basis of phase-space arguments or almost constant form factors. If both the experiments of Brown et al. and Dobbs et al. prove to be correct, the hypothesis of almost constant form factors will no longer be tenable, while the assumption of a partially conserved current may finally attain some degree of experimental confirmation.

As a further application of our hypothesis of a "dominating" K^* and a partially-conserved current, we have computed the form factors for neutral K leptonic decay and have compared our results for $K_2^0 \rightarrow e^+ + \nu + \pi^-$ with the experiment of Luers et al.⁽⁸⁾

If we denote the corresponding form factors for K_2^0 leptonic decay by $h_+(s)$ and $h_-(s)$, we end up with the familiar form

$$h_+(s) = \lambda_2 \left\{ \frac{1}{1 - \frac{s}{M^2}} - 1 + \delta_2 \right\} \quad , \quad (9)$$

$$h_-(s) = - \lambda_2 \frac{\Delta m^2}{M^2} \frac{1}{1 - \frac{s}{M^2}} \quad . \quad (10)$$

If there was only an $I = \frac{1}{2}$ current, then the spectra in K^+ and K_2^0 leptonic decay would be identical. In K_{e3}^+ decay we found that the π^0 energy spectrum had a zero when the kinetic energy of the π^0 was about 85 MeV (see Figure 11). Since such a zero is not observed by Luers et al. in K_{2e3}^0 decay, we must have both $I = 3/2$ and $I = 1/2$ currents. The present data do not allow a useful determination of λ_2 and δ_2 .

It is interesting to note that if there exists a spin 1 $K\pi$ resonance other than the K^* , then irrespective of the isotopic spin of this new particle, the form factors $f_+(s)$, $f_-(s)$, $h_+(s)$, $h_-(s)$ will still have the same effective representations (7), (8), (9), (10) if we neglect quadratic terms in s . Only the physical interpretation of λ , δ , λ_2 , and δ_2 will change. Hence, within the approximations made, our theory is not sensitive to the possible existence of other spin 1 $K\pi$ resonances.

Let us now briefly turn to the leptonic decay of the Λ . There, the strong interaction matrix elements of interest are

$$\begin{aligned} \langle p | s_\mu^V(0) | \Lambda \rangle &= \left(\frac{m_p m_\Lambda}{E_p E_\Lambda} \right)^{\frac{1}{2}} \bar{u}_p \left[i \gamma_\mu F_1(s) + i \frac{1}{2} [\gamma_\mu, \gamma_\nu s_\nu] F_2(s) \right. \\ &\quad \left. + s_\mu F_3(s) \right] u_\Lambda \\ \langle p | s_\mu^A(0) | \Lambda \rangle &= \left(\frac{m_p m_\Lambda}{E_p E_\Lambda} \right)^{\frac{1}{2}} \bar{u}_p \left[i \gamma_\mu \gamma_5 G_1(s) + i \frac{1}{2} [\gamma_\mu, \gamma_\nu s_\nu] \gamma_5 G_2(s) \right. \\ &\quad \left. + \gamma_5 s_\mu G_3(s) \right] u_\Lambda, \end{aligned}$$

where $s_\mu = (p_\Lambda - p_p)_\mu$ and $s = -s_\mu s_\mu$.

We consider the structure of s_μ^V . Proceeding as before, we find that F_1 receives only p-wave contributions and that

$$\langle p | \partial_\alpha s_\alpha^V(0) | \Lambda \rangle \propto \Delta m F_1(s) + s F_3(s) = \omega \Delta m D(s)$$

receives only s-wave contributions. $\Delta m = m_\Lambda - m_p$. Because we do not know of any zero mass particle that would give rise to poles in our

* There is some experimental evidence for the existence of such a resonance with a mass of 730 MeV. See G. Alexander, G. R. Kalbfleisch, D. H. Miller, and G. A. Smith, Phys. Rev. Letters 8, 447 (1962).

form factors, we find

$$F_1(s) = \omega \left\{ D(0) + \frac{s}{M^2} \frac{1}{1 - \frac{s}{M^2}} + V(s) - V(0) \right\} ,$$

$$F_3(s) = - \omega \frac{\Delta m^2}{M^2} \left\{ \frac{1}{1 - \frac{s}{M^2}} + \frac{M^2}{s} (V(s) - V(0)) - \frac{M^2}{s} (D(s) - D(0)) \right\} ,$$

where $V(s)$ represents all p-wave contributions to $F_1(s)$ other than those of the K^* . M is the mass of the K^* , and as in the case of K leptonic decay, we assume that $|D(s)| \ll 1$ in the physical region for s .

The point we wish to stress is that while $F_3(s)$ may be treated as being essentially constant, $F_1(s)$ might be a rapidly varying function of s and may even pass through zero in the physical region. Up to this time, it has been customary to take all form factors constant⁽⁹⁾ and, because of the small momentum transfers involved, $(m_\Lambda - m_p)^2 \leq s \leq m_\ell^2$, the terms containing $F_2(s)$ and $F_3(s)$ have been neglected compared to the term containing $F_1(s)$. It is quite possible that this procedure is not justified.

Once again the concept of a universal Fermi interaction is ill defined because of the rapid variation of $F_1(s)$. As in leptonic K decay, an explanation for the slowness of the vector part of $\Lambda\beta$ -decay may be connected with the partial conservation of s_Q^V . Because of the lack of experimental evidence and the wealth of unknown constants in the form factors, we are not able to say more about the problem at this time.

FIGURE CAPTIONS FOR K LEPTONIC DECAY

Fig. 1: The branching ratio $K_{\mu 3}^+/K_{e 3}^+$ is plotted as a function of the parameter δ . The experimental value for the branching ratio of 0.96 ± 0.15 is represented by the horizontal solid and dashed lines. This indicates that the range of δ is limited to $-0.1 \leq \delta \leq -0.025$. The sharp rise is due to the zero in the form factor $f_+(s)$ which suppresses the $K_{e 3}^+$ rate more than the $K_{\mu 3}^+$ rate because $K_{e 3}^+$ depends only on $f_+(s)$, while $K_{\mu 3}^+$ depends on both $f_+(s)$ and $f_-(s)$.

Fig. 2: The form factor $f_+(s)$ is given for three values of the parameter δ within the range determined by the branching ratio $K_{\mu 3}^+/K_{e 3}^+$. The coupling constant λ has been divided out of $f_+(s)$.

Fig. 3: The effective coupling constant squared, λ^2 , is plotted as a function of δ . The experimental rate of $4.0 \times 10^6 \text{ sec}^{-1}$ for $K_{e 3}^+$ decay has been used. The dashed vertical lines indicate the restriction placed on δ by the known $K_{\mu 3}^+/K_{e 3}^+$ branching ratio.

Fig. 4: The K leptonic decay rates are given as a function of δ with λ set equal to 1.

Fig. 5: The histogram gives the π energy spectrum in the $K_{\mu 3}^+$ decay as measured by Brown et al.. The kinetic energy of the π is T_π . The smooth theoretical curves have been corrected for experimental biases. Brown et al. use a two-parameter fit, while the theory proposed in this paper uses the one-parameter δ . The curve labeled $\xi = -9$ is the constant

form factor theory implied by the experiments of Dobbs et al. and Boyarski et al.

Fig. 6: The histogram gives the μ^+ energy spectrum in the $K_{\mu 3}^+$ decay as measured by Brown et al. The kinetic energy of the μ is T_μ . The smooth theoretical curves have been corrected for experimental biases. Brown et al. use a two-parameter fit, while the theory proposed in this paper uses the one-parameter δ .

Fig. 7: The experimental μ^+ energy spectrum, as measured by Dobbs et al., is represented by the histogram. The histogram has been corrected for experimental biases.

Fig. 8: The π energy spectrum predicted by various theories is given. Note the sensitivity of the spectrum to values of δ . The dip in the spectrum for the curve $\delta = -0.065$ is due to the zero of $f_+(s)$ in the physical region of s .

Fig. 9: The μ^+ energy spectrum predicted by various theories is given.

Fig. 10: The longitudinal polarization of the μ in $K_{\mu 3}^+$ decay is plotted as a function of μ kinetic energy. Although the polarization fluctuates wildly with small changes in δ , large μ energies always yield negative polarizations.

Fig. 11: The size of effects due to a vector boson X mediating the weak interactions is given for the electron energy spectrum in the $K_{e 3}^+$ decay. Although strong interactions could give rise to similar variations in the electron spectrum, the zero in the spectrum is a definite peculiarity of our theory arising from the zero of $f_+(s)$ in the physical region of s .

A direct measurement of this spectrum would be a crucial test for the hypothesis of a partially conserved current.

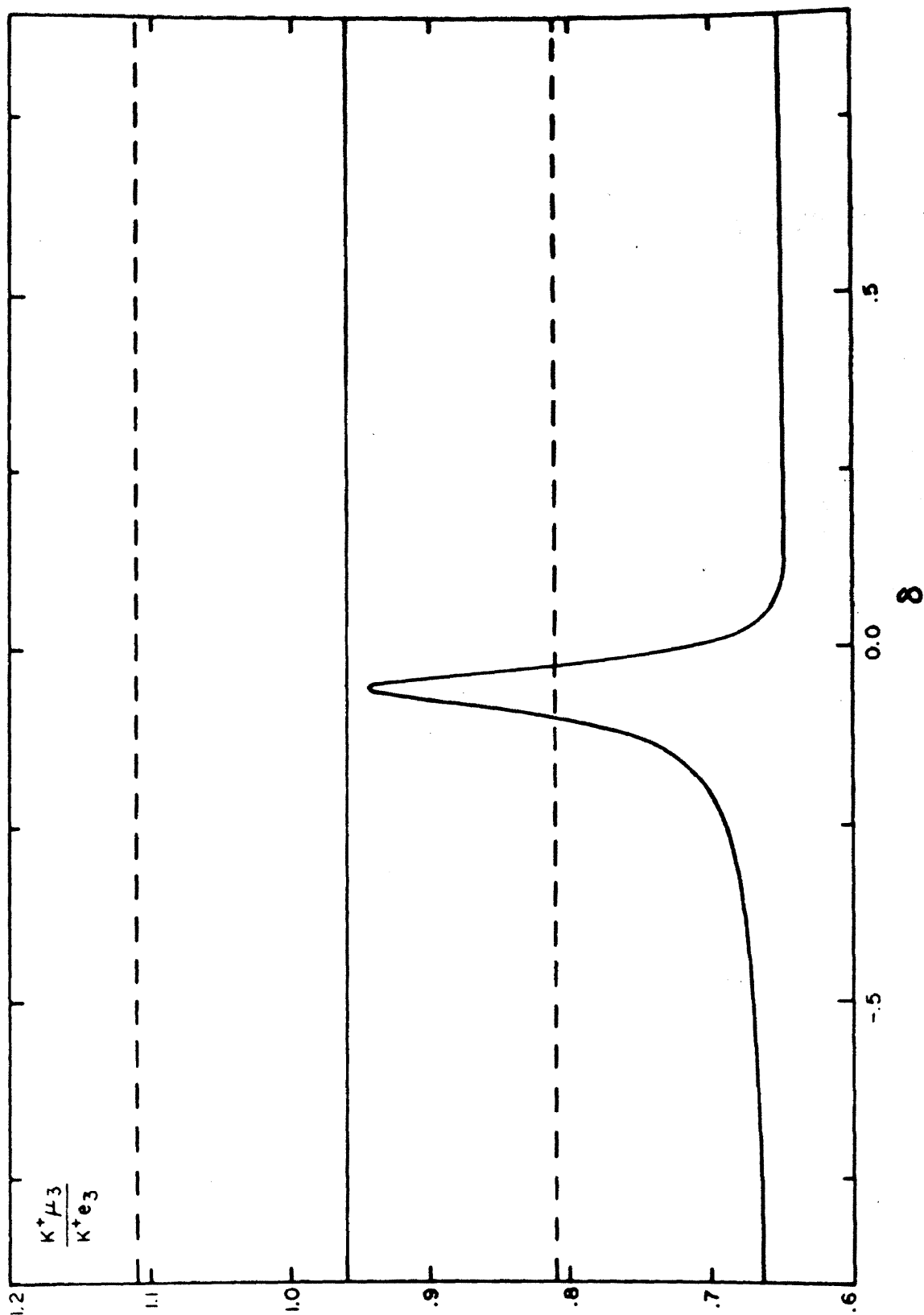


Figure 1

$\lambda^{-1} f_+(s)$

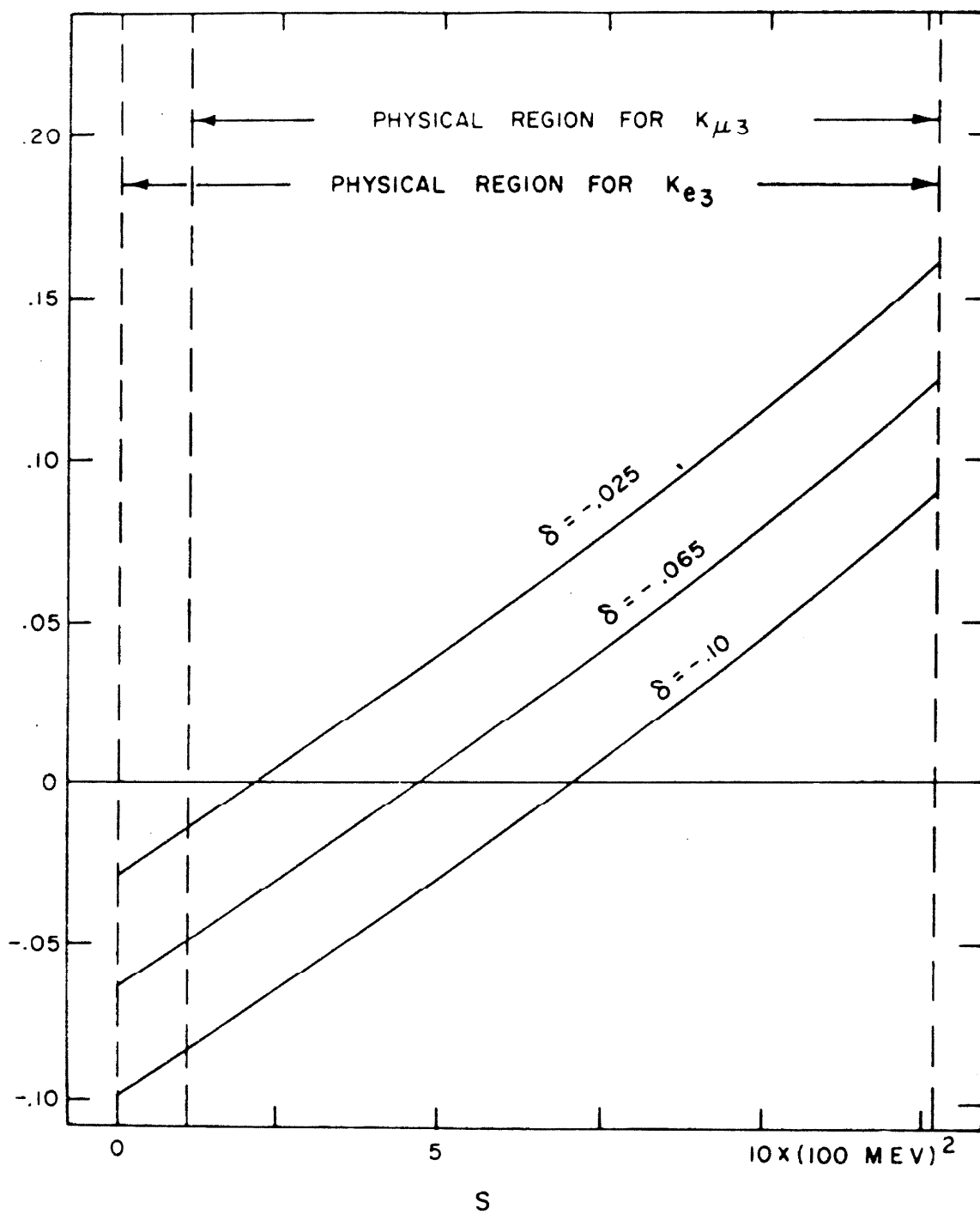


Figure 2

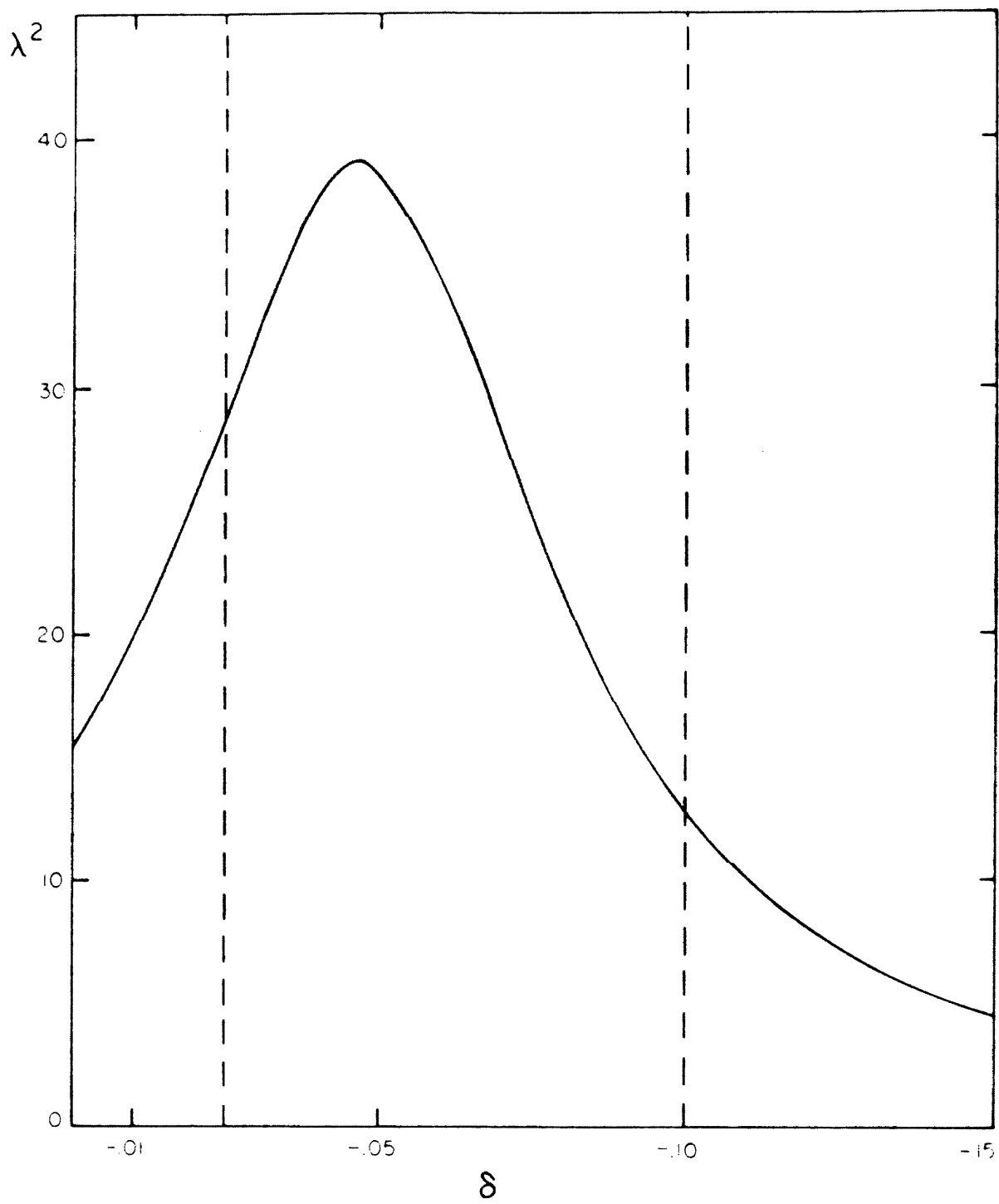


Figure 3

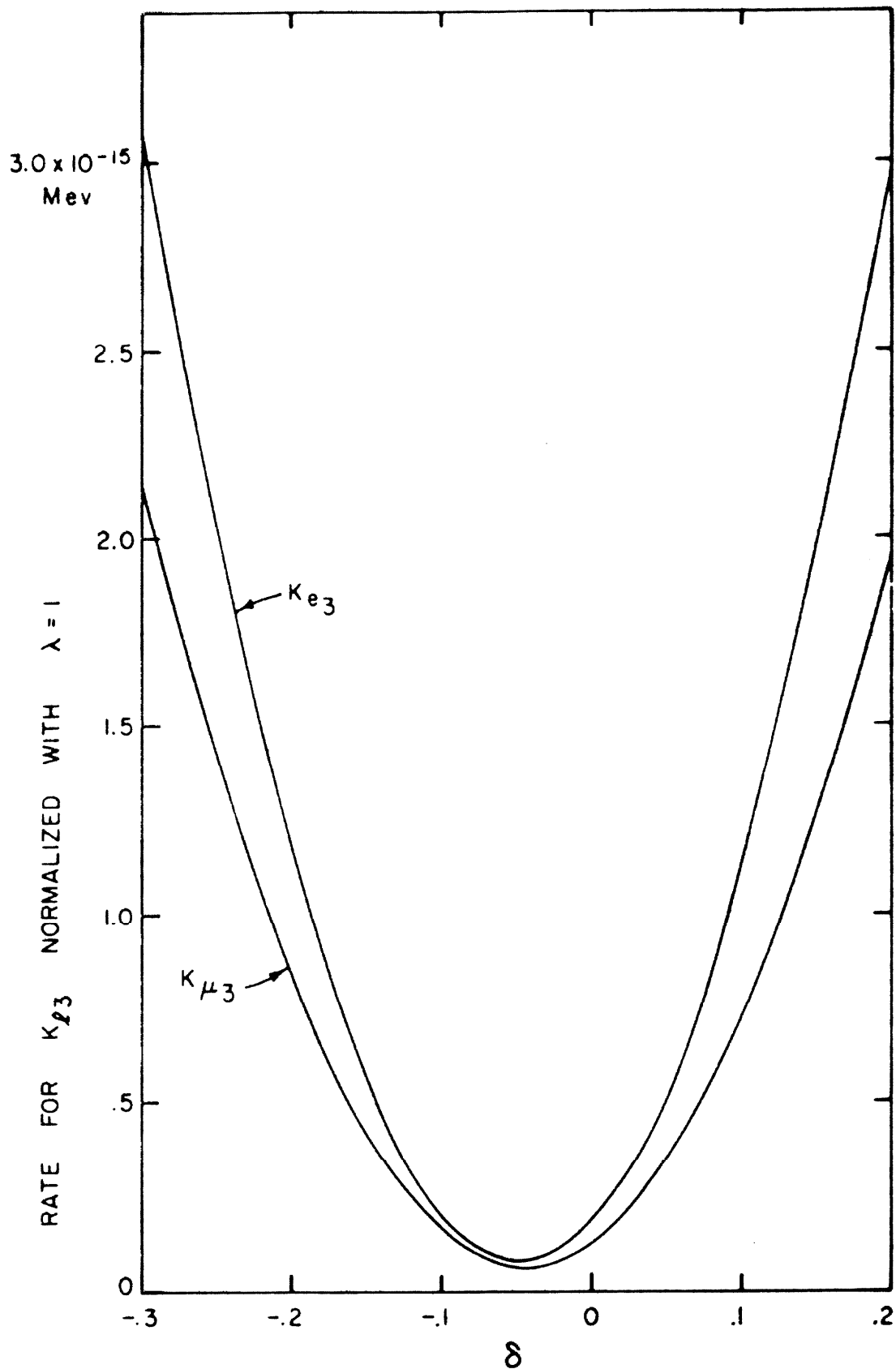


Figure 4

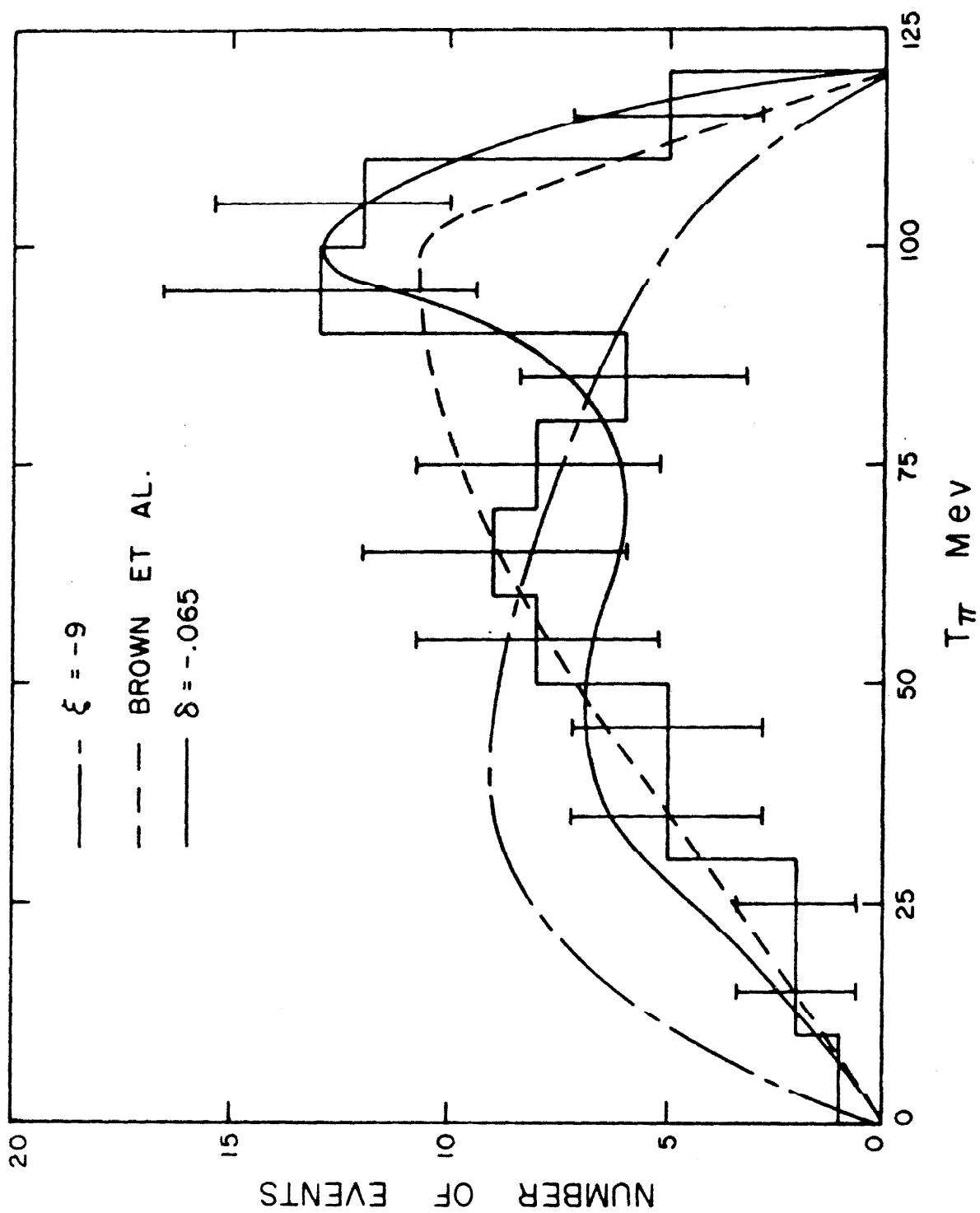


Figure 5

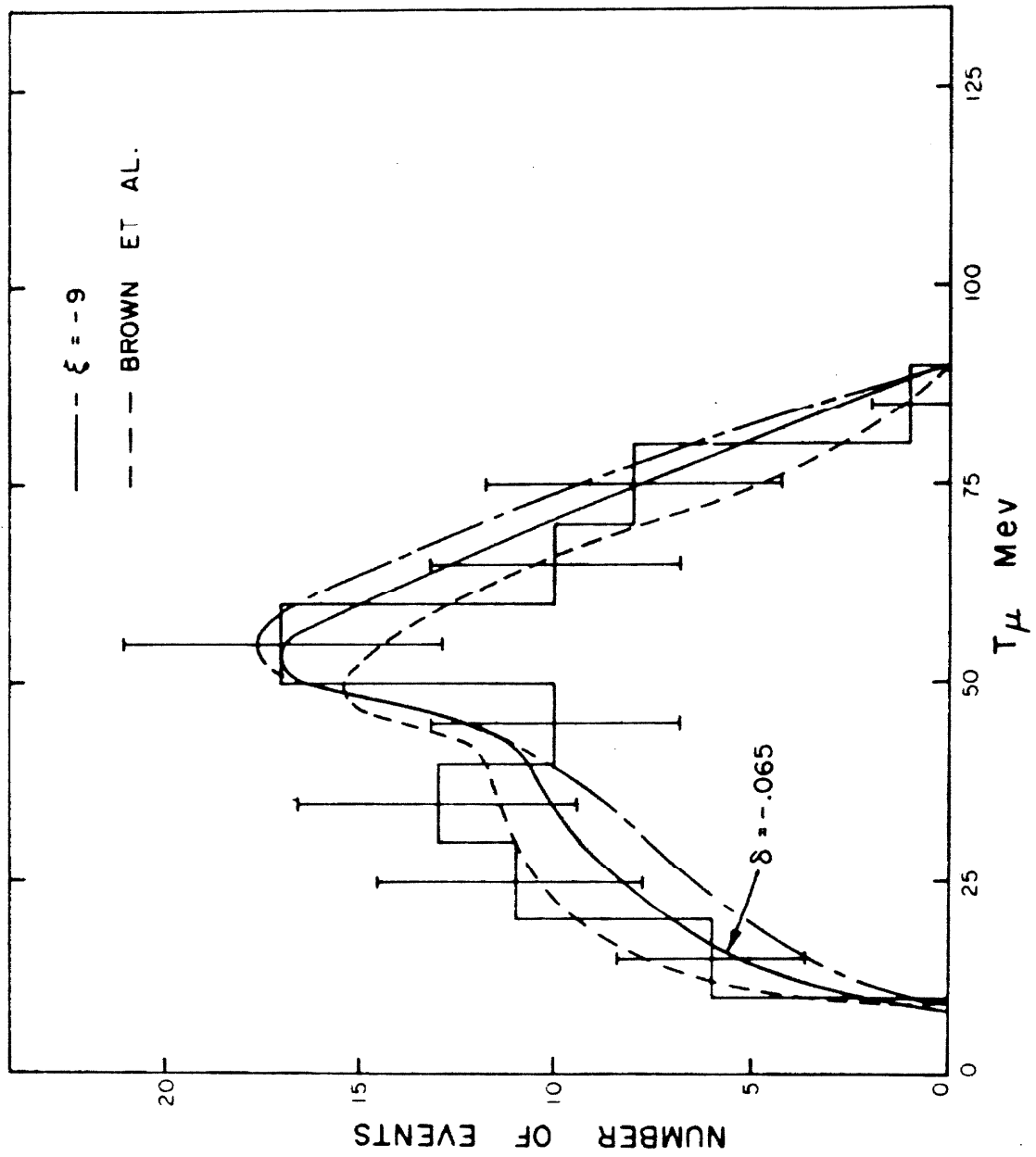


Figure 6

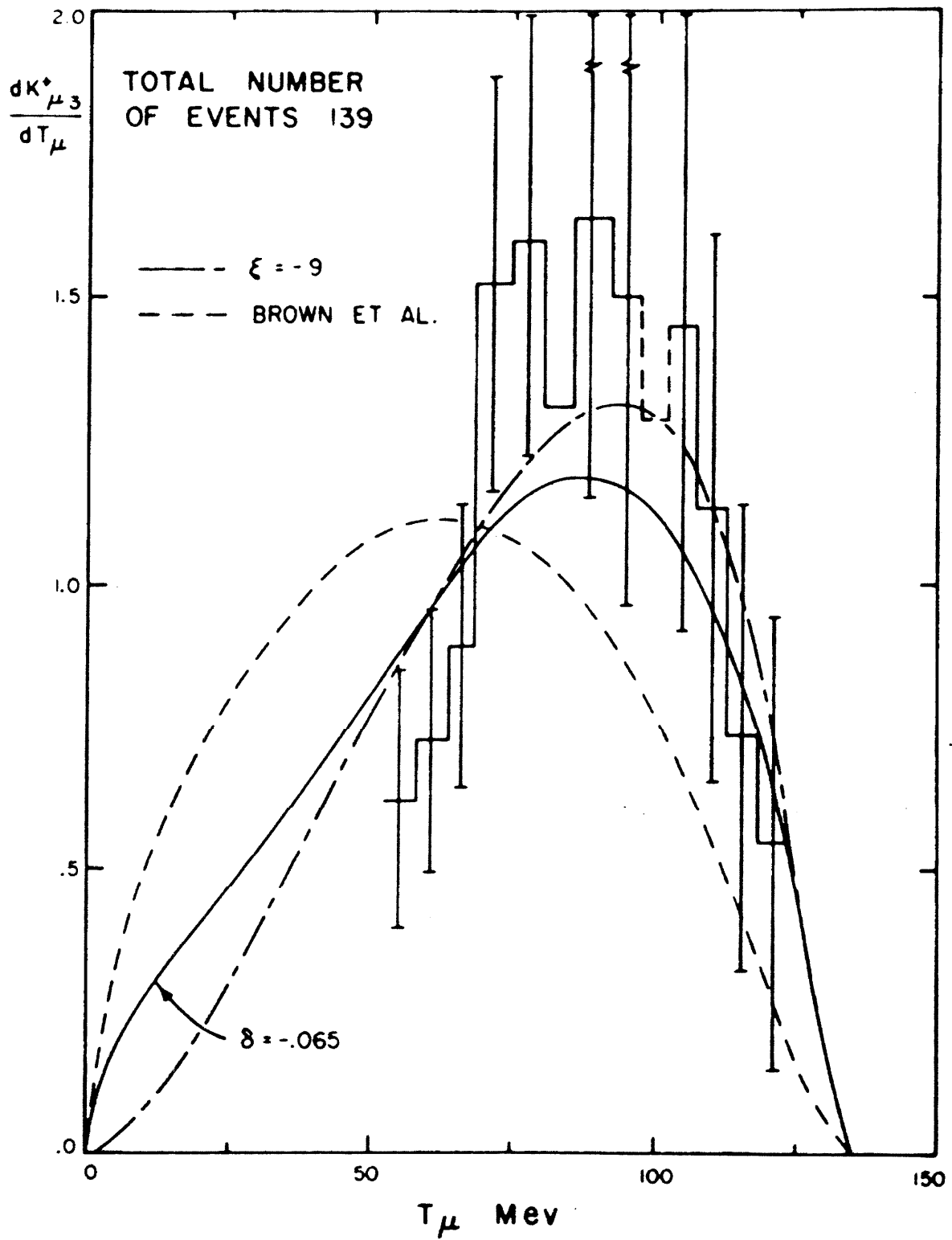


Figure 7

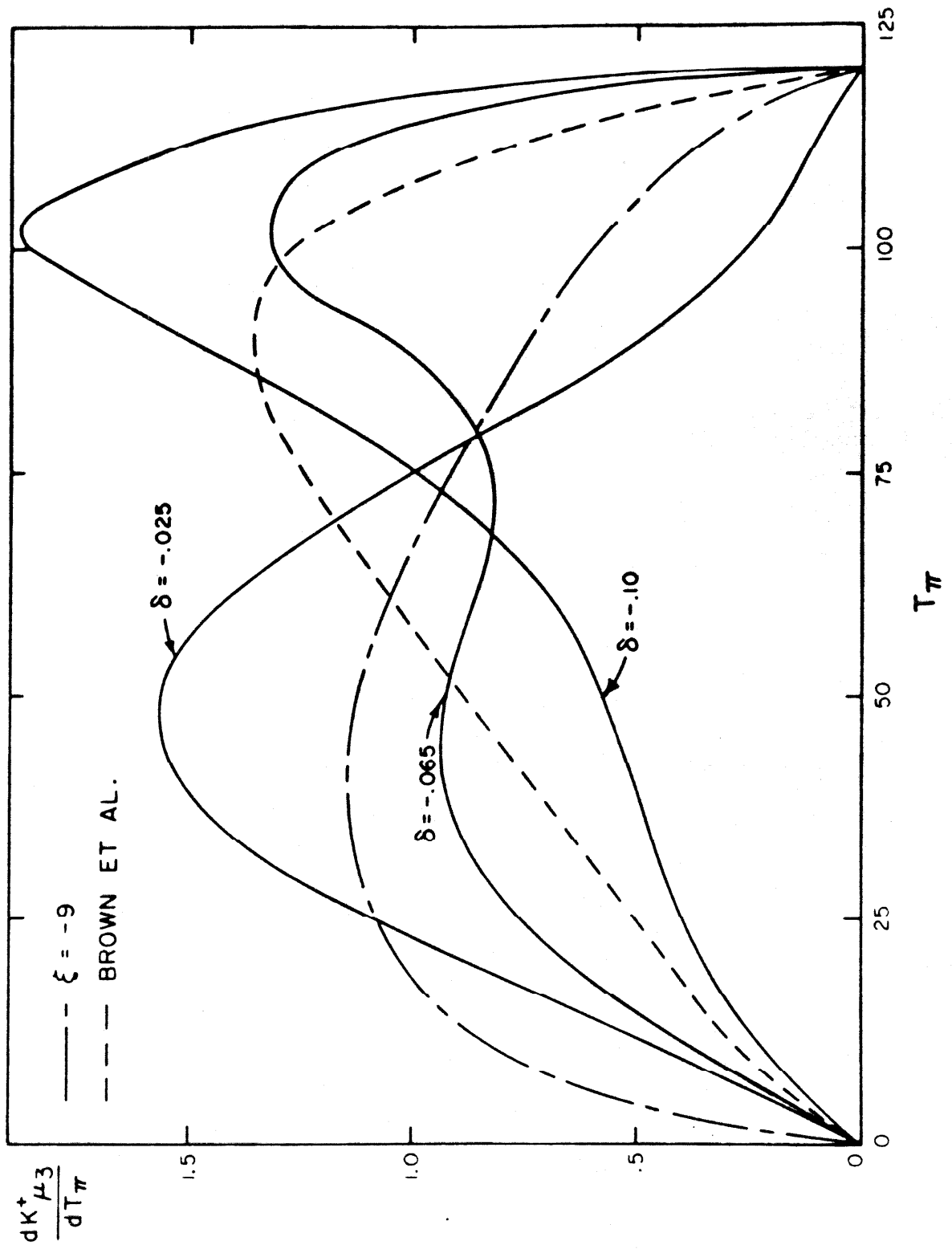


Figure 8

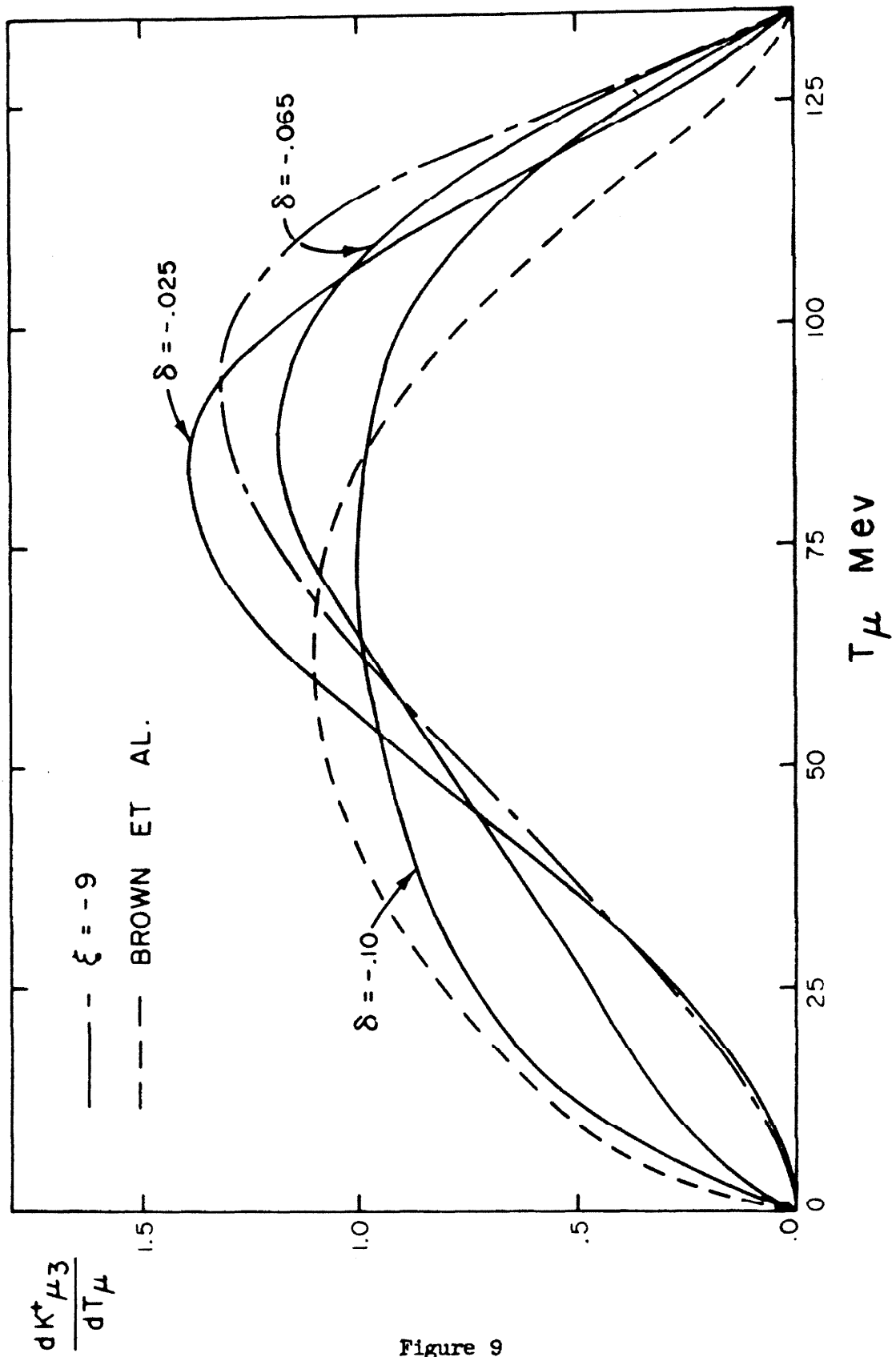


Figure 9

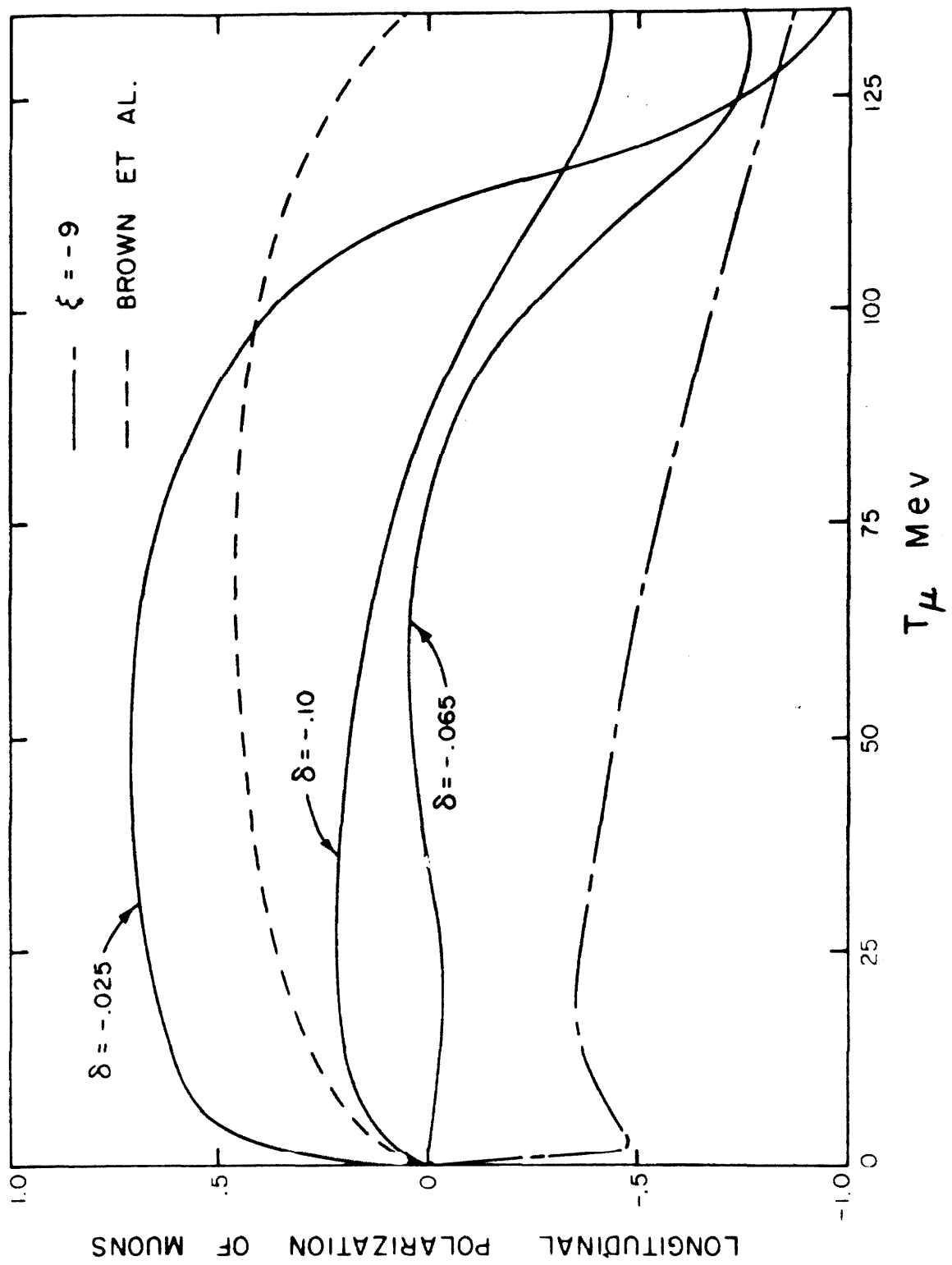
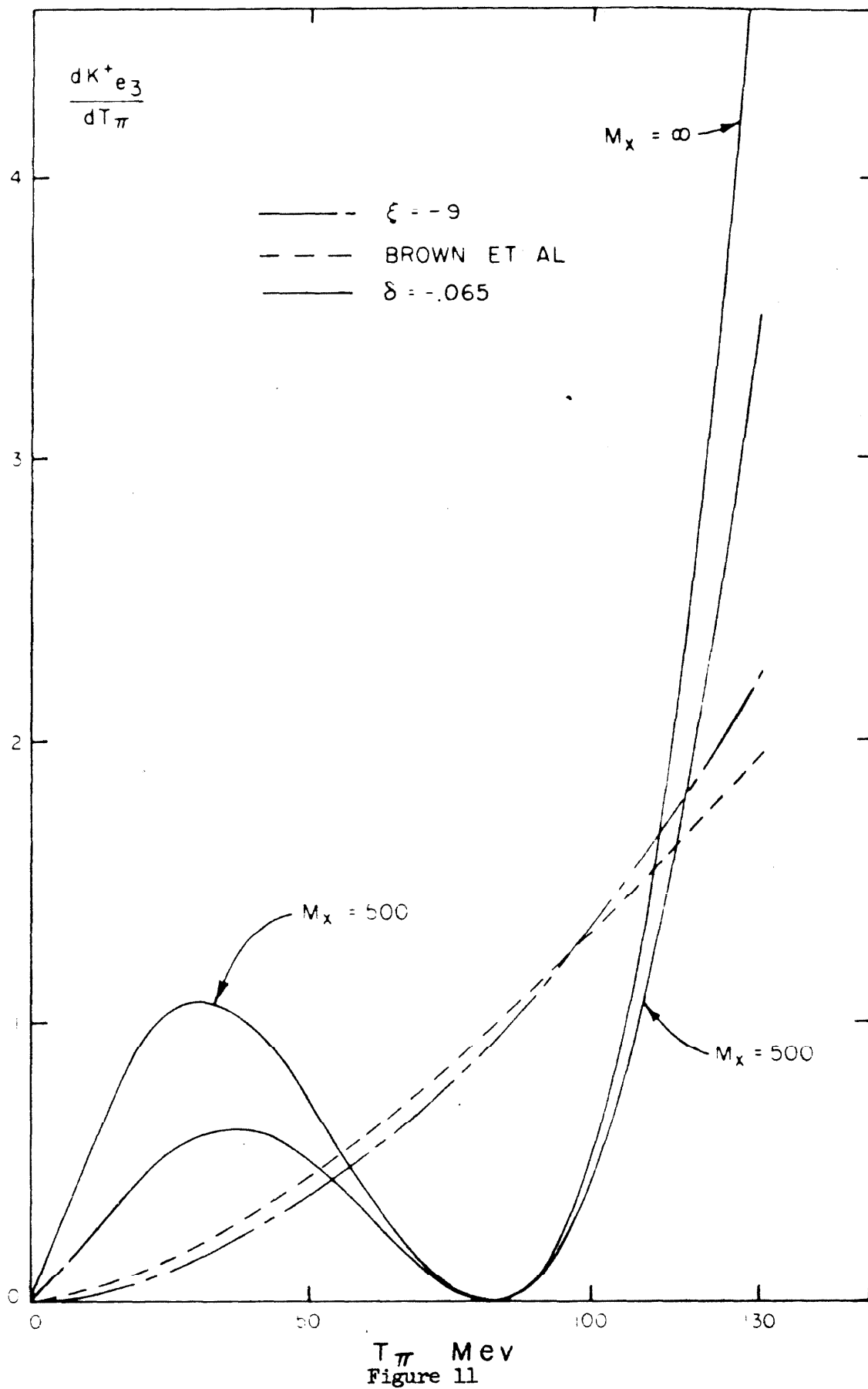


Figure 10



REFERENCES FOR $\gamma + N \rightarrow \pi + N$

1. Murray Gell-Mann, Phys. Rev. Letters 8, 263 (1962).
2. V. H. Gribov and I. Ya. Pomeranchuk, Phys. Rev. Letters 8, 343 (1962).
3. W. G. Wagner, Phys. Rev. Letters 10, 202 (1963).
4. T. F. Kycia and K. F. Riley, Phys. Rev. Letters 10, 266 (1963).
5. G. Hohler and K. Dietz, Institut fur Theoretische Kernphysik der Technischen Hochschule-Karlsruhe preprint March 1963.
6. G. F. Chew, M. L. Goldberger, F. E. Low, Y. Nambu, Phys. Rev. 106, 1345 (1957).
7. James Stutsman Ball, Phys. Rev. 124, 2014 (1961).
8. James Stutsman Ball, UCRL Report 8858 (unpublished).
9. M. Jacob and G. C. Wick, Ann. Phys. 7, 404 (1959).
10. M. Gordin, D. Lurie, and A. Martin; Nuovo Cimento 18, 933 (1960).
11. G. F. Chew and S. C. Frautschi Phys. Rev. Letters 7, 394 (1961).
12. G. F. Chew and S. C. Frautschi Phys. Rev. Letters 8, 41 (1962).
13. S. C. Frautschi, M. Gell-Mann, and F. Zachariasen, Phys. Rev. 126, 2203 (1962).
14. M. Gell-Mann, Phys. Rev. Letters 8, 263 (1962).
15. B. M. Udgaonkar, Phys. Rev. Letters 8, 142 (1962).
16. C. E. Jones and J. A. Poirer, UCRL Report 10677 (unpublished).

REFERENCES FOR K LEPTONIC DECAY

1. J. Bernstein and S. Weinberg, Phys. Rev. Letters 5, 481 (1960);
H. Chew, Phys. Rev. Letters 8, 297 (1962).
2. J. Bernstein, S. Fubini, M. Gell-Mann, and W. Thirring, Nuovo
Cimento 17, 757 (1960).
3. D. H. Sharp and W. G. Wagner, California Institute of Technology
Synchrotron Laboratory Report CTSL-34 (1962) (unpublished).
4. See, e.g., S. Okubo, Nuovo Cimento 13, 292 (1959).
5. See, e.g., J. Bernstein and S. Weinberg, loc. cit.
6. J. L. Brown, J. A. Kadyk, G. H. Trilling, R. T. Van de Walle,
B. P. Roe, and D. Sinclair, Phys. Rev. Letters 8, 450 (1962).
7. J. M. Dobbs, K. Lande, A. K. Mann, K. Reibel, F. J. Sciulli,
H. Uto, D. H. White, and K. K. Young, Phys. Rev. Letters 8,
295 (1962); A. M. Boyarski, E. C. Loh, L. Q. Niemela,
D. M. Ritson, R. Weinstein, and S. Ozaki, Phys. Rev.
128, 2398 (1962).
8. D. Luers, I. S. Mittra, W. J. Willis, and S. S. Yamamoto, Phys.
Rev. Letters 7, 255 (1961).
9. See, e.g., R. Norton, Phys. Rev. 126, 1216 (1962).
10. See especially N. Brene, L. Egardt, B. Qvist, and D. A. Geffen,
Nuclear Physics 30, 399 (1962).